Karl Keating's "Scientific" Attempt to Debunk Geocentrism

http://forums.catholic.com/showthread.php?t=851392&page=2

Re: Mic'd Up (Robert Sungenis and the Principle movie)

R. Sungenis: Karl Keating, president of *Catholic Answers*, has attempted to debunk geocentrism on scientific grounds. On his *Catholic Answers* forum, Mr. Keating answered a moniker by the name of "buffalo." In this paper I will show why Mr. Keating's answer is wrong.

Buffalo: "The math works either way. Science leaves us with two options: geocentricity or acentricity. Pick a worldview. It is looking like Galileo was wrong."

Keating: The math may work either way, but the physics doesn't. This is a distinction the proponents of geocentrism fail to see. In fact, their whole argument relies on this distinction being overlooked.

R. Sungenis: Keating's statement is wrong. If the math of either system works, it is because the physics of either system works, for physics is measuring how things move by using mathematics, not intuition or magic.

Keating: Simple example: If we have two celestial bodies, A and B, orbiting one another, we can posit that A is stationary and that B orbits it. We can work up an equation that allows us to plot B's path around A. On the other hand, we can assume B is stationary and can plot an equally simple equation that allows us to plot A's path around B. So far no problem, until you get to the real world and ask, Why are these bodies orbiting one another? If A is a star and B is a planet, we need to explain why these bodies don't fly away from one another. Why do they remain as a "system"? The answer, of course, is gravity. And this get us past mere math and into physics.

R. Sungenis: Again, there is no such thing as "mere math" in physics. Physics is little more than math. If the math doesn't work, then neither will the physics. The problem with physics is that it can provide more than one viable math solution, and different math solutions yield different physical explanations. Picking the right physical explanation is often beyond the bounds of physics.

Keating: If one of these bodies is stationary and is orbited by the other body, the stationary body must exert enough gravity to hold the orbiting body in its circulating path. If the gravitation pull is too weak, the orbiting body will fly away.

R. Sungenis: This is essentially correct, but still a little naïve. Here is an example. Venus is orbiting the sun. If the sun were to decrease in mass then its gravitational pull would be less. Venus would then increase its orbital diameter, not fly away. Venus would only fly away if the sun were to become so small that there was little mutual gravitational attraction between it and the sun.

Keating: It should be clear that a star can "capture" a planet but that a planet has too little gravity to force a star away from continuing on a straight path and to force it to stay on an orbiting path.

R. Sungenis: Yes, that is correct. And the math would tell us that the planet does not have a strong enough force of gravity to capture the star. In cases like this Newton's equation $F = m_1m_2/r^2$ is used to make the calculation.

Keating: So, yes, the math "works" either way, when we're talking about two points or weightless bodies orbiting one another: either one could be the stationary center. But the situation changes when we start to talk about actual things such as the Sun and the Earth.

R. Sungenis: Here is where Keating runs off the tracks. Notice how Keating seeks to limit that issue to "the Sun and the Earth." If the issue were limited to the Sun and the Earth, Mr. Keating would be correct. That is, the Earth, being the smaller body, would necessarily orbit the Sun, which is the larger body. This is precisely what led Galileo to surmise that if small moons are orbiting Jupiter, then the smaller Earth should orbit the sun, and thus the Earth moves. So, contrary to Mr. Keating's implication, geocentrists are well aware of how classical physics deals with this issue.

Where Mr. Keating goes wrong is precisely his attempt to limit the issue to a two-body system, the Sun and the Earth. I'm sure Mr. Keating has noticed that each night we see that there are countless stars the circle the Earth. Each of those 5 sextillion stars have gravity, and that gravity will affect how the Sun and Earth react to one another, especially if the Earth is put in the center of that gravity.

But let's not take my word for it. Let's look at what one famous astrophysicist said about the issue. Sir Fred Hoyle put it this way:

> Let it be understood at the outset that it makes no difference, from the point of view of describing planetary motion, whether we take the Earth or the Sun as the center of the solar system. Since the issue is one of relative motion only, there are infinitely many exactly equivalent descriptions referred to different centers – in principle any point will do, the Moon, Jupiter….So the passions loosed on the world by the publication of Copernicus' book, *De revolutionibus orbium caelestium libri VI*, were logically irrelevant…[1]
>
> …we can take either the Earth or the Sun, or any other point for that matter, as the center of the solar system. This is certainly so for the purely kinematical problem of describing the planetary motions. It is also possible to take any point as the center even in dynamics, although recognition of this freedom of choice had to await the present century.[2]

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[1] Fred Hoyle, *Nicolaus Copernicus: An Essay on his Life and Work*, p. 1. Two years later he wrote: "We know that the difference between a heliocentric theory and a geocentric theory is one of relative motion only, and that such a difference has no physical significance. But such an understanding had to await Einstein's theory of gravitation in order to be fully clarified" (*Astronomy and Cosmology*, 1975, p. 416).

[2] Fred Hoyle, *Nicolaus Copernicus: An Essay on his Life and Work*, p. 82. Also from the same book: "Today we cannot say that the Copernican theory is "right" and the Ptolemaic theory is "wrong" in any meaningful sense. The two theories are…physically equivalent to one another" (*ibid*, p. 88). Physicist J. L. McCauley who reviewed Hoyle's book stated it was "The only brief account, using understandable modern terminology, of what Ptolemy and Copernicus really did. Epicycles are just data analysis (Fourier series), they don't imply any underlying theory of mechanics. Copernicus did not prove that the Earth moves, he made the

Now, notice that Hoyle said that it is not just in the geometric sense that any point can be made the center, but also in the "dynamic" sense. What is "dynamics"? Nothing more than gravity and the movements of bodies within that gravity.

Notice also that Hoyle said that modern science didn't know about this truth until "the present century," which, when he was writing, was the 20th century, four centuries after Newton's original gravitational equations.

So, we've certainly come a long way since Newton. Newton's laws work fine if we limit the components to two bodies, but when we have three, four or billions of them, Newton's laws are quite limited in their scope and need to be supplemented. Newton was supplemented by Mach and Einstein.

Incidentally, modern science has discovered that, if they heed to their Big Bang theory, Newton's laws hardly work at all in outer space. For example, it has been discovered that the spin rate of spiral galaxies is ten times too fast to fit into Newton's laws. So in order to compensate for the needed matter (since Newton's laws are based on the amount of matter present), modern science invents the matter it needs and calls it "Dark Matter" (which shows us how precarious the modern Big Bang theory is). Additionally, the Big Bang needs more matter in its original explosion to account for the creation of galaxies, but since the theory shows there wasn't enough baryonic matter, modern science again says that "Dark Matter" was in the original Big Bang explosion and helped create the galaxies.

Back to our issue. We have found that limiting the problem to "the Sun and the Earth" (as Mr. Keating tried to do) is not going to work. At the least, it is not going to prove anything for Mr. Keating.

Let's look at another statement from Hoyle to get a bigger picture of the issue:

> Although in the nineteenth century this argument [e.g., Keating's argument] was believed to be a satisfactory justification of the heliocentric theory, one found causes for disquiet if one looked into it a little more carefully. When we seek to improve on the accuracy of calculation by including mutual gravitational interactions between planets, we find – again in order to calculate correctly – that the center of the solar system must be placed at an abstract point known as the "center of mass," which is displaced quite appreciably from the center of the Sun. And if we imagine a star to pass moderately close to the solar system, in order to calculate the perturbing effect correctly, again using the inverse-square rule, it could be essential to use a "center of mass" which included the star. The "center" in this case would lie even farther away from the center of the Sun. It appears, then, that the "center" to be used for any set of bodies depends on the way in which the local system is considered to be isolated from the universe as a whole. If a new body is added to the set from outside, or if a body is taken away, the "center" changes.[3]

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equivalent of a coordinate transformation and showed that an Earth-centered system and a sun-centered system describe the data with about the same number of epicycles. For the reader who wants to understand the history of ideas of motion, this is the only book aside from Barbour's far more exhaustive treatment" (Letters on File, 10-1-04).

[3] Fred Hoyle, *Nicolaus Copernicus*, 1973, p. 85.

So we see that when the stars are added to the equation, things change quite drastically. In fact, we can envision a universe of stars spaced all over the sphere of the universe, and somewhere in the middle of all those stars will be a "center of mass" around which those stars will revolve. Logically, there is no reason why the Earth cannot occupy that center of mass. If the Earth occupies the center of mass, then according to Newton's laws, there are no gravitational or inertial forces at that point, and thus there is no force with which the Sun needs to interact. The Earth is neutral. Hence, contrary to Mr. Keating's view, we do not have a "two-body" issue and we cannot use F = m1m2/r^2 to calculate which body is orbiting, the Sun or the Earth. We can certainly grant to Mr. Keating that, with local systems that are far away from the universe's center of mass, it will always be the case that the smaller revolves around the larger, such as is the case of smaller moons orbiting the larger planet Jupiter. But if we include the whole universe, then there is one place in which the larger will revolve around the smaller. The smaller, in this case, is at the universe's center of mass, which the Earth occupies.

Does modern science allow this arrangement? We have seen thus far that Newton can have no argument with it. His only problem was that in the 1600s when he developed his gravitational equations, he didn't realize what part the stars and the rest of the universe played in the calculations.

As for Einstein's equations, which are merely a "relativistic" expansion of Newton's equations, they perfectly agree with the idea that the whole universe can revolve around a fixed Earth in the center.

> We need not necessarily trace the existence of these centrifugal forces back to an absolute movement of K' [Earth]; we can instead just as well trace them back to the rotational movement of the distant ponderable masses [stars] in relation to K' whereby we treat K' as 'at rest.'…On the other hand, the following important argument speaks for the relativistic perspective. The centrifugal force that works on a body under given conditions is determined by precisely the same natural constants as the action of a gravitational field on the same body (*i.e.*, its mass), in such a way that we have no means to differentiate a 'centrifugal field' from a gravitational field….This quite substantiates the view that we may regard the rotating system K' [Earth] as at rest and the centrifugal field as a gravitational field….The kinematic equivalence of two coordinate systems, namely, is not restricted to the case in which the two systems, K [the universe] and K' [the Earth] are in uniform relative translational motion. The equivalence exists just as well from the kinematic standpoint when for example the two systems rotate relative to one another.[4]

In other words, Einstein's equations state that either the Earth can rotate in a non-rotating universe or the universe can rotate around a non-rotating Earth. The math AND the physics will allow such variation. The problem for General Relativity is that it can't tell us which one is correct. And that's where we come in ☺

Here's another quote from Max Born. He is referencing the work of Hans Thirring who used Einstein's equations to prove what Einstein stated above.

[4] Einstein's October 1914 paper titled: "Die formale Grundlage der allgemeinen Relativitätstheorie," trans. by Carl Hoefer, in *Mach's Principle: From Newton's Bucket to Quantum Gravity*, eds. Julian Barbour and Herbert Pfister, pp. 69, 71.

> ...Thus we may return to Ptolemy's point of view of a 'motionless Earth.' This would mean that we use a system of reference rigidly fixed to the Earth in which all stars are performing a rotational motion with the same angular velocity around the Earth's axis…one has to show that the transformed metric can be regarded as produced according to Einstein's field equations, by distant rotating masses. This has been done by Thirring. He calculated a field due to a rotating, hollow, thick-walled sphere and proved that inside the cavity it behaved as though there were centrifugal and other inertial forces usually attributed to absolute space. Thus from Einstein's point of view, Ptolemy and Copernicus are equally right. What point of view is chosen is a matter of expediency.[5]

In other words, Thirring showed that the inertial forces we experience from a rotating Earth (e.g., the Corilois force that turns the Foucault Pendulum), can also be attributed to a universe rotating around a fixed Earth. Which one we choose as correct is based on other grounds, and normally that ground is philosophical or theological.

In even more modern times, extended equations have been produced which show that the universe can revolve around a fixed Earth at the center. This one is from Dr. Luka Popov and was published in the *European Journal of Physics* in 2013.

> The analysis of planetary motions has been performed in the Newtonian framework with the assumption of Mach's principle. The kinematical equivalence of the Copernican (heliocentric) and the Neo-tychonian (geocentric) systems is shown to be a consequence of the presence of pseudo-potential in the geocentric system, which, according to Mach, must be regarded as the real potential originating from the fact of the simultaneous acceleration of the Universe. This analysis can be done on any other celestial body observed from the Earth. Since Sun and Mars are chosen arbitrarily, and there is nothing special about Mars, one can expect to come up with the same general conclusion. There is another interesting remark that follows from this analysis. If one could put the whole Universe in accelerated motion around the Earth, the pseudo-potential corresponding to pseudo-force will immediately be generated. That same pseudo-potential causes the Universe to stay in that very state of motion, without any need of exterior forces acting on it.[6]

There are many more perspectives and equations to show the viability of the geocentric system from a dynamic point of view. These are all included in our book, Galileo Was Wrong. I invite Mr. Keating and anyone else on his forum to obtain the book and see for themselves.

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[5] Max Born, *Einstein's Theory of Relativity*, 1962, 1965, pp. 344-345. Thirring's model has been duplicated by Barbour & Bertotti (*Il Nuovo Cimento B*, 38:1, 1977) and Joseph Rosen ("Extended Mach's Principle," *American Journal of Physics*, Vol 49, No. 3, March 1981) using Hamiltonians; and by William G. V. Rosser (*An Introduction to the Theory of Relativity*, 1964) who expanded on Thirring's paper and and noted that the universe's rotation can exceed *c* by many magnitudes; Christian Møller (*The Theory of Relativity*, 1952) who also extended Thirring's paper using a ring universe rather than a shell; G. Burniston Brown ("A Theory of Action at a Distance," *Proceedings of the Physical Society*, 1955) who discovered geocentrism based on Newtonian physics; Parry Moon and Domina Spencer ("Mach's Principle," *Philosophy of Science*, 1959) who arrive at geocentrism using Mach's principle; J. David Nightingale ("Specific physical consequences of Mach's principle," 1976) who transposed the Einstein equation of Mach's principle into Newtonian physics for a geocentric universe; and several others do the same.

[6] Luka Popov, "Newtonian–Machian analysis of the neo-Tychonian model of planetary motions," *European Journal of Physics*, 34, 383-391 (2013). Also available at arXiv:1301.6045 [physics.class-ph]. Dr. Popov is employed by the Dept. of Physics, University of Zagreb, Bujenička cesta 32, Zagreb, Croatia.

Now, for those who really want to see the details of the math, allow me to give you what is presently in Galileo Was Wrong, the 9th edition. In the upcoming 10th edition, we include the calculation of the geocentric Lagrange Points using potentials.

2. Two-Body Problem in the Central Potential

2.1 General overview

We start with the overview of two body problem in Newtonian mechanics. Although there are alternative and simpler ways to solve this problem,[7] we will follow the usual textbook approach.[8] The Lagrangian of the system reads:

$$L = \tfrac{1}{2}m_1\dot{\mathbf{r}}_1^2 + \tfrac{1}{2}m_2\dot{\mathbf{r}}_2^2 - U(|\mathbf{r}_1 - \mathbf{r}_2|) \qquad (2.1)$$

where U is potential energy that depends only on the magnitude of the difference of radii vectors (so-called central potential). We can easily rewrite this equation in terms of relative position vector $\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$, and let the origin be at the center of mass, *i.e.*, $m_1\mathbf{r}_1 + m_2\mathbf{r}_2 \equiv 0$. The solution of these equations is:

$$\mathbf{r}_1 = \frac{m_2}{m_1 + m_2}\mathbf{r}, \qquad \mathbf{r}_2 = -\frac{m_2}{m_1 + m_2}\mathbf{r}\,. \qquad (2.2)$$

The Lagrangian (2.1) so becomes

$$\mathrm{L} = \tfrac{1}{2}\mu\dot{\mathbf{r}}^2 - U(r) \qquad (2.3)$$

where $\mathbf{r} \equiv |\mathbf{r}|$ and μ is the reduced mass,

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} \qquad (2.4)$$

In that manner, the two-body problem is reduced to one body problem of particle with coordinate **r** and mass μ in the potential $U(r)$.

Using polar coordinates, the Lagrangian (3) can be written as:

$$\mathrm{L} = \tfrac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2) - U(\mathrm{r}) \qquad (2.5)$$

One can immediately notice that variable ϕ is cyclic (it does not appear in the Lagrangian explicitly). Consequence of that fact is momentum conservation law, since $(\partial/\partial t)\,(\partial L/\partial\phi) = \partial L/\partial\phi = 0$. Therefore,

$$\ell \equiv \frac{\partial L}{\partial \phi} = \mu r^2\dot{\phi} = \text{const.} \qquad (2.6)$$

is the integral of motion.

In order to find a solution for the trajectory of a particle, it is not necessary to explicitly write down the Euler-Lagrange equations. Instead, one can use the energy conservation law,

[7] Hauser, W., 1985, "On planetary motion," *Am. J. Phys*., 53, 905–7; Gauthier, N., 1986, "Planetary orbits," *Am. J. Phys*., 54, 203.
[8] Landau, L. D. and Lifshiz, E. M., 1976, *Mechanics*, 3rd edn., Oxford: Butterworth-Heinemann, pp. 25–40; Goldstein, H., 1980, *Classical Mechanics*, 2nd edn., Reading, MA: Addison-Wesley, pp. 70–102.

$$E = \tfrac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2) + U(r) = \tfrac{1}{2}\mu\dot{r}^2 + \frac{\ell^2}{2\mu r^2} + U(r) \quad (2.7)$$

Straightforward integration of (2.7) gives the equation for the trajectory,

$$\phi(r) = \int \frac{\ell\, \mathrm{d}r/r^2}{\sqrt{2m[E - U(r) - \ell^2/r^2}} \quad (2.8)$$

2.2 **Kepler's problem**

Let us now consider the particle in the potential

$$U(r) = -\frac{k}{r} \quad (2.9)$$

generally known as *Kepler's problem*. Since our primary interest is in the planetary motions under the influence of gravity, we will take $k > 0$. Solution of eq. (8) for that potential is:

$$\frac{p}{r} = 1 + e \cos \phi, \quad (2.10)$$

where $2p$ is called the *lactus rectum* of the orbit, and e is the eccentricity. These quantities are given by

$$p = \frac{2\ell^2}{\mu k}, \qquad e = \sqrt{1 + \frac{2E\ell^2}{\mu k^2}} \quad (2.11)$$

Expression (2.10) is the equation of a conic section with one focus in the origin. For $E < 0$ and $e < 1$ the orbit is an ellipse.

One can also determine minimal and maximal distances from the source of the potential, called perihelion and aphelion, respectively:

$$r_{min} = \frac{p}{1+e}, \qquad r_{max} = \frac{p}{1-e} \quad (2.12)$$

These parameters can be directly observed, and often are used to test a model or a theory regarding planetary motions.

3. Earth and Mars in the Heliocentric Perspective

According to Newton's law of gravity, the force between two massive objects reads:

$$\mathbf{F} = \frac{Gm_1m_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}(\mathbf{r}_1 - \mathbf{r}_2) \quad (3.1)$$

Which leads to a potential ($\mathbf{F} = -\nabla U$)

$$U(|\mathbf{r}_1 - \mathbf{r}_2|) = -\frac{Gm_1m_2}{|\mathbf{r}_1 - \mathbf{r}_2|} \quad (3.2)$$

This is obviously Kepler's potential (2.9) with $k = Gm_1m_2$, where G is Newton's gravitational constant.

Since the Sun is more than 5 orders of magnitude more massive than Earth and Mars, we will in all future analysis use the approximation

$$\mu \approx m_i \tag{3.3}$$

where m_i is mass of the observed planet. For the same reason, gravitational interaction between Earth and Mars can be neglected, since it is negligible compared with the interaction between Mars and the Sun. Using these assumptions, we can write down corresponding Lagrangians,

$$L_{ES} = \tfrac{1}{2} m_E \dot{\mathbf{r}}_{ES}^2 + \frac{G m_E M_S}{\mathbf{r}_{ES}},$$

$$L_{MS} = \tfrac{1}{2} m_M \dot{\mathbf{r}}_{MS}^2 + \frac{G m_M M_S}{\mathbf{r}_{MS}} \tag{3.4}$$

where m_E and m_M are masses of Earth and Mars, respectively. Subscripts *ES* (*MS*) correspond to the motion of Earth (Mars) with respect to the Sun. These trajectories can be calculated using the exact solution (2.10) with appropriate strength constants k and initial conditions which determine E and ℓ. Another way is to solve the Euler-Lagrange equations numerically, using astronomical parameters[9] (*e.g.*, aphelion and perihelion of Earth/Mars) to choose the initial conditions that fit the observed data. The former has been done using *Wolfram Mathematica* package. The result is shown on Fig. 1.

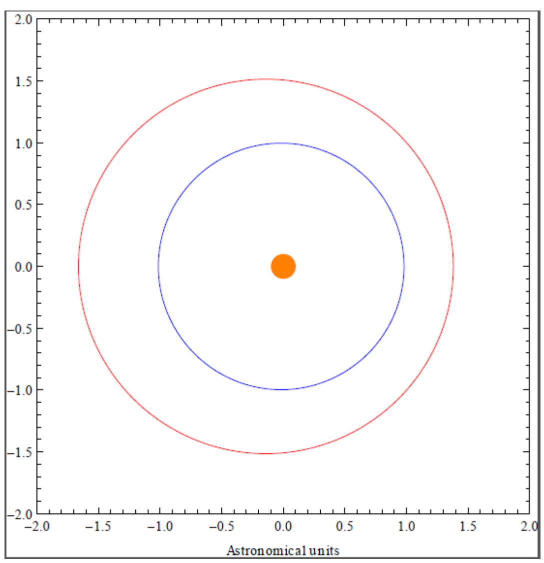

FIG. 1: Trajectories of Earth and Mars in heliocentric system over the period of 2 years. Blue and red lines represent Earth's and Mars' orbits, respectively.

For the later comparison, one could write out the expressions for the *e* and *p* parameters for the Earth. Putting the expressions for energy (2.7) and momentum (2.6) into eqs. (2.11) it is straightforward to obtain

[9] Weast, R. C. (ed), 1968, *Handbook of Chemistry and Physics*, 49th edn., Cleveland, OH: Chemical Rubber Company, pp. F145–6.

$$p = \frac{\dot{\phi}^2 r^4}{GM_S}$$

$$e = \sqrt{1 - \frac{2GM_S\dot{\phi}^2 r^3 - \dot{r}^2\dot{\phi}^2 r^4 - \dot{\phi}^4 r^6}{G^2 M_S^2}} \tag{3.5}$$

where ϕ, $\dot{r}$ and r are angular velocity, radial velocity and distance respectively, taken in the same moment of time (*e.g.* in $t = 0$).

Fig. 2 displays motion of the Mars as viewed from the Earth, gained by trivial coordinate transformation

$$\mathbf{r}_{EM}(t) = -\mathbf{r}_{ES}(t) + \mathbf{r}_{MS}(t), \tag{3.6}$$

where $\mathbf{r}_{ES}(t)$ and $\mathbf{r}_{MS}(t)$ are solutions of Euler-Lagrange equations for the Lagrangians (3.4). Equation (3.6) is just the mathematical expression of the Tycho Brahe's claim. The retrograde motion of Mars can be useful in the attempt to understand and determine orbital parameters, as was shown qualitatively and quantitatively by Thompson.[10]

The acceleration that Earth experiences due to the gravitational force of the Sun is usually referred as centripetal acceleration and is given by

$$\mathbf{a}_{cp} = \frac{\mathbf{F}_{cp}}{m_E} = \frac{GM_S}{r_{ES}^2}\hat{\mathbf{r}}_{ES} \tag{3.7}$$

where $\hat{\mathbf{r}}$ is the unit vector in the direction of vector $\mathbf{r}$, $\mathbf{r}_{ES}(t)$ is radius vector describing motion of Earth around the Sun, and F_{cp} is centripetal force, *i.e.* the force that causes the motion.

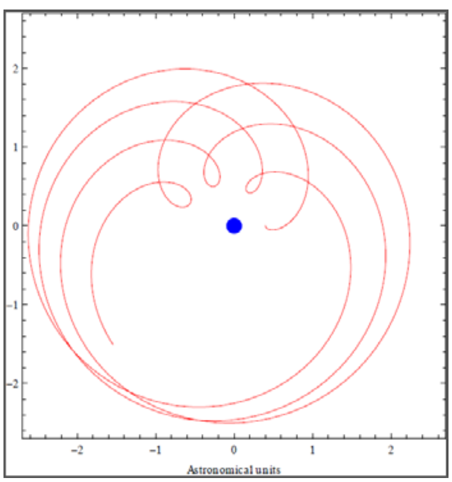

FIG. 2: Trajectory of the Mars as seen from the Earth over the period of 7 years. Calculation of this trajectory is done numerically in the heliocentric system.

4. Sun and Mars in the Geocentric Perspective

4.1 The pseudo-potential

From the heliocentric perspective, the fact that the Earth moves around the Sun results with centrifugal pseudo-force, observed only by the observer on the Earth. But if we apply Mach's principle to the geocentric viewpoint, one is obliged to speak about the real forces resulting from

[10] Thompson, B. G. 2005, "Using retrograde motion to understand and determine orbital parameters," *Am. J. Phys*., 73, 1023–9.

the fact that the Universe as a whole moves around the observer on the stationary Earth. Although these forces will further be considered as the real forces, we well keep the usual terminology and call them pseudo-forces, for the sake of convenience. Our focus here will be on the annual orbits, not on diurnal rotation which requires some additional physical assumptions[11] that are beyond the scope of this paper.

The Universe is regarded as an $(N + 1)$–particle system (N celestial bodies plus planet Earth). From the point of a stationary Earth, one can write down the Lagrangian that describes the motions of celestial bodies:

$$L = ½\sum_{i=1}^{N} m_i\, \dot{r}_i^2 - ½\sum_{i=1}^{N} \frac{Gm_i m_j}{r_{ij}} - \sum_{i=1}^{N} \frac{Gm_E m_i}{r_i} - U_{ps}, \tag{4.1}$$

where $r_{ij} \equiv |\mathbf{r}_i - \mathbf{r}_j|$, U_{ps} stands for the pseudo-potential, satisfying $\mathbf{F}_{ps} = -\nabla U_{ps}$. $\mathbf{F}_{ps}$ is the pseudo-force given by

$$\mathbf{F}_{ps} = -m\sum_{i=1}^{N} \mathbf{a}_{cp,i}\,, \tag{4.2}$$

where $\mathbf{a}_{cp,i}$ is centripetal acceleration for given celestial body (with respect to the Earth) and m is a mass of the object that is subjected to this force. It's easy to notice that the dominant contribution in these sums comes from the Sun. The close objects (planets, moons, etc.) are much less massive than the Sun, and massive objects are much farther distant. The same approximation is implicitly used in section 3.

In the Machian picture, the centripetal acceleration is a mere relative quantity, describing the rate of change of relative velocity. Therefore, centripetal acceleration of the Sun with respect to Earth is given by Equation 3.7, with $\mathbf{r}_{ES} = -\mathbf{r}_{SE}$. All that considered, Equation 4.2 becomes

$$\mathbf{F}_{ps} = \frac{GmM_S}{r_{SE}^2}\,\hat{\mathbf{r}}_{SE} \tag{4.3}$$

where $\mathbf{r}_{SE}(t)$ describes the motion of the Sun around the Earth.

We can now finally write down the pseudo-potential which influences every body observed by the fixed observer on Earth:

$$U_{ps}(\mathbf{r}) = \frac{Gm\,M_S}{r_{SE}^2}\,\hat{\mathbf{r}}_{SE} \cdot \mathbf{r} \tag{4.4}$$

where $\mathbf{r}(t)$ describes motion of particle of mass m with respect to the Earth. Notice that this is not a central potential.

4.2 The Sun in Earth's pseudo-potential

In order to determine Sun's orbit in Earth's pseudo-potential, one needs to take the dominant contributions of the Lagrangian (4.1), as was explained earlier. Taking into account the expression for pseudo-potential given in Equation 4.4, one ends up with

$$L_{SE} = ½M_S\dot{r}_{SE}^2 - \frac{GM_S^2}{r_{SE}} \tag{4.5}$$

[11] Vetö, B., 2011, "Gravitomagnetic field of the universe and Coriolis force on the rotating Earth," *Eur. J. Phys.*, 32, 1323–9; Assis, A. K. T., 1999, *Relational Mechanics*, Montreal: Aperion.

This Lagrangian has the exact same form as the reduced Lagrangian (2.3). That means that we can immediately determine the orbit by means of Equation (2.11) by substituting $\mu = M_S$ and $k = GM_S^2$. This leads to the following result (subscript *SE* will be omitted):

$$p = \frac{\dot{\phi}^2 r^4}{GM_S}$$

$$e = \sqrt{1 - \frac{2GM_S\dot{\phi}^2 r^3 - \dot{r}^2\dot{\phi}^2 r^4 - \dot{\phi}^4 r^6}{G^2 M_S^2}} \qquad (4.6)$$

which is the exact equivalent of the previous result given in Equations (3.5), since $\dot{\phi}$, $\dot{r}$ and r are relative quantities, by definition equivalent in both models. We can therefore conclude that the Sun's orbit in the Earth's pseudo-potential is equivalent as one observed from the Earth in the heliocentric system. It remains to show the same thing for Mars' orbit.

4.3 Mars in Earth's pseudo-potential

In the similar way as before, we take the dominant contributions of Lagrangian (4.1) together with Equation (4.4) and form the Lagrangian:

$$L_{ME} = ½ m_M \dot{\mathbf{r}}_{ME}^2 + \frac{Gm_M M_S}{|\mathbf{r}_{ME} - \mathbf{r}_{SE}|} - \frac{Gm_M M_S}{r_{SE}^2} \hat{\boldsymbol{r}}_{SE} \cdot \boldsymbol{r}_{ME} \qquad (4.7)$$

where subscript *ME* refers to the motion of Mars with respect to Earth, and $\mathbf{r}_{SE}(t)$ is solution of the Euler-Lagrange equations for the Lagrangian (4.5).

The Euler-Lagrange equations for $\boldsymbol{r}_{ME}(t)$ Lagrangian (4.7) are too complicated to be solved analytically, but can easily be solved numerically. The numerical solutions for equations of motion for both the Sun and Mars are displayed in Fig. 3. The equivalence of trajectories gained in two different ways is obvious, justifying the model proposed by Tycho Brahe.

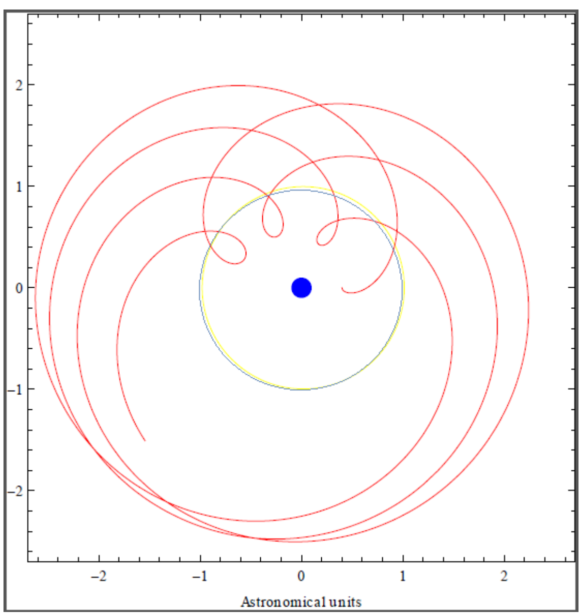

FIG. 3: Trajectories of the Sun (dark, blue) and the Mars (light, red) moving in Earth's pseudo-potential over the period of 7 years. Calculation of this trajectory is performed numerically in the geocentric system.

5. Conclusion

The analysis of planetary motions has been performed in the Newtonian framework with the assumption of Mach's principle. The kinematical equivalence of the Copernican (heliocentric) and the Neo-tychonian (geocentric) systems is shown to be a consequence of the presence of a pseudo-potential (4.4) in the geocentric system, which, according to Mach, must be regarded as the real potential originating from the fact of the simultaneous acceleration of the Universe. This analysis can be done on any other celestial body observed from the Earth. Since Sun and Mars are chosen arbitrarily, and there is nothing special about Mars, one can expect to come up with the same general conclusion.

There is another interesting remark that follows from this analysis. If one could put the whole Universe in accelerated motion around the Earth, the pseudo-potential corresponding to the pseudo-force (4.2) will immediately be generated. That same pseudo-potential then causes the Universe to stay in that very state of motion, without any need of exterior forces acting on it. See the following.

The Dynamical Description of the Geocentric Universe

Using Mach's principle, we will show that the observed diurnal and annual motion of the Earth can just as well be accounted as the diurnal rotation and annual revolution of the Universe around the fixed and centered Earth. This can be performed by postulating the existence of vector and scalar potentials caused by the simultaneous motion of the masses in the universe, including the distant stars.

1. Introduction

The modern day use of the word relativity in physics is usually connected with Galilean and special relativity, *i.e.*, the equivalence of the systems performing the uniform rectilinear motion, so-called inertial frames. Nevertheless, the physicists and philosophers never ceased to debate the various topics under the heading of Mach's principle, which essentially claims the equivalence of all co-moving frames, including non-inertial frames as well.

Historically, this issue was first brought out by Sir Isaac Newton in his famous rotating bucket argument. As Newton saw it, the bucket is rotating in the absolute space and that motion produces the centrifugal forces manifested by the concave shape of the surface of the water in the bucket. The motion of the water is therefore to be considered as "true and absolute," clearly distinguished from the relative motion of the water with respect to the vessel.[12]

Mach, on the other hand, called the concept of absolute space a "monstrous conception,"[13] and claimed that the centrifugal force in the bucket is the result only of the relative motion of the water with respect to the masses in the Universe. Mach argued that if one could rotate the whole Universe around the bucket, the centrifugal forces would be generated, and the concave-shaped surface of the water in the bucket would be identical as in the case of rotating bucket in the fixed Universe. Mach extended this principle to the once famous debate between geocentrists and heliocentrists, claiming that both systems can equally be considered correct.[14]

His arguments, however, remained mostly of a philosophical nature. Since he was a convinced empiricist, he believed that science should be operating only with observable facts, and the only thing we can observe is relative motion. Therefore, every notion of absolute motion or a preferred

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[12] Newton, I., 1960, *The Mathematical Principles of Natural Philosophy*, Berkeley CA: University of California Press, pp 10-11.
[13] Mach, E., 1960, *The Science of Mechanics*, 6th ed., LaSalle IL: Open Court, p. xxviii.
[14] *Ibid*., pp. 279, 284.

inertial frame, whether inertial or non-inertial, is not a scientific one but rather a mathematical or philosophical preference.

As Hartman and Nissim-Sabat correctly point out,[15] Mach never formulated the mathematical model or an alternative set of physical laws which can explain the motions of the stars, the planets, the Sun and the Moon in a Tychonian or Ptolemaic geocentric systems. For that reason, some physicists in modern times have tried to "Machianize" the Newtonian mechanics in various ways[16] [17] or even try to construct new theories of mechanics.[18] There have also been attempts to reconcile Mach's principle with the General Theory of Relativity, some of which were profoundly analyzed in the paper by Raine.[19]

In the recent paper[20] we have used the concept of the so-called pseudo-force and derived the expression for the potential which is responsible for it. This potential can be considered as a real potential (as shown by Zylbersztajn,[21] which can easily explain the annual motion of the Sun and planets in the Neo-Tychonian system. In the same manner, one can explain the annual motion of the stars and the observation of the stellar parallax.[22]

It is the aim of this paper to use the same approach to give the dynamical explanation of the diurnal motion of the celestial bodies as seen from the Earth, and thus give the mathematical justification for the validity of Mach's arguments regarding the equivalence of the Copernican and geocentric systems. The paper is organized as follows. In section 2 the vector potential is introduced in general terms. This formalism is then applied to analyze the motions of the celestial bodies as seen from the Earth in section 3. Finally, the conclusion of the analysis is given.

2. Vector potential formalism

Following Mach's line of thought, one can say that the simultaneously rotating Universe generates some kind of gravito-magnetic vector potential, **A**. By the analogy with the classical theory of fields[23] one can write down the Lagrangian which includes the vector potential,

$$L = \frac{1}{2}m\dot{\mathbf{r}}^2 + \mathrm{m}\dot{\mathbf{r}} \cdot \mathbf{A} + \frac{1}{2}m\mathbf{A}^2 - mU_{ext} \tag{2.1}$$

where m is the mass of the particle under consideration, and U_{ext} is some external scalar potential imposed on the particle, for example, the gravitational interaction.

We know, as an observed fact, that every body in the rotational frame of reference undergoes the equations of motion given by[24]

$$\mathrm{m}\ddot{\mathbf{r}} = \mathbf{F}_{\mathrm{ext}} - 2m(\boldsymbol{\omega}_{\mathrm{rel}} \times \dot{\mathbf{r}}\,) - m[\boldsymbol{\omega}_{\mathrm{rel}} \times (\boldsymbol{\omega}_{\mathrm{rel}} \times \mathbf{r})] \tag{2.2}$$

where ω_{rel} is the relative angular velocity between the given frame of reference and the distant masses in the Universe, and $\mathbf{F}_{\mathrm{ext}} = -\nabla U_{\mathrm{ext}}$ some external force acting on a particle.

It can be easily demonstrated that one can derive Equation (2.2) by applying the Euler-Lagrange equations on the following "observed" Langrangian

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[15] Hartman, H. I., and Nissim-Sabat, C., 2003, *Am. J. Phys*. 71, 1163–68.
[16] Hood, C. G. 1970, *Am. J. Phys*. 38, 438–442.
[17] Barbour, J., 1974, *Nature* 249, 328–329.
[18] Assis, A. K. T., 1999, *Relational Mechanics*, Montreal: Aperion.
[19] Raine, D. J., 1981, *Rep. Prog. Phys*. 44, 1151–95.
[20] Popov, L., 2013, *Eur. J. Phys*., 34, 383–391, Corrigendum, 2013, *Eur. J. Phys*. 34, 817, Preprint: arXiv:1301.6045.
[21] Zylbersztajn, A., 1994, *Eur. J. Phys*. 15, 1–8.
[22] Popov, L., 2013, arXiv:1302.7129.
[23] Landau, L. D. and Lifshiz, E. M., 1980, *The Classical Theory of Fields*, 4th ed., Oxford: Butterworth-Heinemann, p. 49.
[24] Goldstein, H., 1980, *Classical Mechanics*, 2nd ed., Reading, MA: Addison-Wesley, p. 178.

$$L_{obs} = \frac{1}{2}m\dot{\mathbf{r}}^2 + m\dot{\mathbf{r}}\cdot(\boldsymbol{\omega}_{rel}\times\mathbf{r}) + \frac{1}{2}m(\boldsymbol{\omega}_{rel}\times\mathbf{r})^2 - mU_{ext} \quad (2.3)$$

By comparison of the general Lagrangian (2.1) and the "observed" Lagrangian (2.3) one can write down the expression for the vector potential **A**,

$$\mathbf{A} = \boldsymbol{\omega}_{rel}\times\mathbf{r} \quad (2.4)$$

It is important to notice that there is no notion of the absolute rotation in this formalism. The observer sitting on the edge of the Newton's rotating bucket can only observe and measure the relative angular velocity between him or her and the distant stars ω_{rel}, incapable of determining whether it is the bucket or the stars that is rotating.

3. Trajectories of the celestial bodies around the fixed Earth

3.1. Diurnal motion

It is one thing to postulate that rotating masses in the Universe generate the vector potential given by (2.4), but quite another to claim that this same potential can be used to explain and understand the very motion of these distant masses. We will now demonstrate that this is indeed the case.

The observer sitting on the surface of the Earth makes several observations. First, he or she notices that there is a preferred axis (say z) around which all the Universe rotates with the period of approximately 24 hours. Then, according to the formalism given in Section 2, he or she concludes that the Earth must be immersed in the vector potential given by

$$\mathbf{A} = \Omega\hat{\mathbf{z}}\times\mathbf{r} \quad (3.1)$$

where $\Omega \approx (2\pi/24\text{h})$ is the observed angular velocity of the celestial bodies.[25]

One can now re-write the Lagrangian (2.1) together with the Equation (3.1) and focus only on the contributions coming from the vector potential **A**,

$$L_{rot} = \frac{1}{2}m\dot{\mathbf{r}}^2 + m\Omega\dot{\mathbf{r}}\cdot(\hat{\mathbf{z}}\times\mathbf{r}) + \frac{1}{2}m\Omega^2(\hat{\mathbf{z}}\times\mathbf{r})^2 \quad (3.2)$$

The Euler-Lagrange equations for this Lagrangian, written for each component of the Cartesian coordinates, are given by

$$\begin{aligned}\ddot{x} &= -2\Omega\dot{y} + \Omega^2 x\\ \ddot{y} &= 2\Omega\dot{x} + \Omega^2 y\\ \ddot{z} &= 0\end{aligned} \quad (3.3)$$

The solution of this system of differential equations reads

$$\begin{aligned}x(t) &= r\cos\Omega t\\ y(\text{t}) &= r\sin\Omega t\\ z(t) &= 0\end{aligned} \quad (3.4)$$

[25] The period of the relative rotation between the Earth and the distant stars is called sidereal day and it equals 23 h 56' 4.0916". Common time on a typical clock measures a slightly longer cycle, accounting not only for the Sun's diurnal rotation but also for the Sun's annual revolution around the Earth (as seen from the geocentric perspective) of slightly less than 1 degree per day (Wikipedia, 26 Apr. 2013, Sidereal time http://http://en.wikipedia.org/wiki/Sidereal_time).

where r is the initial distance of the star from the z axis. The observer can therefore conclude that the celestial bodies perform real circular orbits around the static Earth due to the existence of the vector potential **A** given by Equation (3.1). This conclusion is equivalent to the one that claims that the Earth rotates around the z axis and the celestial bodies do not.

3.2. Annual motion

The second thing the observer on the Earth notices is the periodical annual motion of the celestial bodies around the z' axis which is inclined form the axis of diurnal rotation z by the angle of approximately 23.5°. This motion can be explained if one assumes that the Earth is immersed in the so-called pseudo-potential

$$U_{\mathrm{ps}}(\mathbf{r}) = \frac{GM_S}{r_{SE}^2}\, \hat{\mathbf{r}}_{\mathrm{SE}} \cdot \mathbf{r} \tag{3.5}$$

Here G stands for Newton's constant, M_S stands for the mass of the Sun and $\mathbf{r}_{\mathrm{SE}}\,(t)$ describes the motion of the Sun as seen from the Earth. The Sun's trajectory $\mathbf{r}_{\mathrm{SE}}\,(t)$ is shown to be an ellipse in x'-y' plane (defined by the z' axis from the above). Using this potential alone one can reproduce the observed retrograde motion of the Mars or explain the effect of the stellar parallax as the real motion of the distant stars in the x'-y' plane. All this was demonstrated in the previous communications [9, 11].

3.3. Total account

One can finally conclude that all celestial bodies in the Universe perform the twofold motion around the Earth:

i. circular motion in the x-y plane due to the vector potential **A** (3.1) with the period of approximately 24 hours, and

ii. elliptical orbital motion in the x'-y' plane due to the scalar potential U_{ps} (3.5) with the period of approximately one year.

Using Equations (2.1), (3.1) and (3.5) one can write down the complete classical Lagrangian of the geocentric Universe,

$$L = \frac{1}{2} m\dot{\mathbf{r}}^2 + m\Omega\dot{\mathbf{r}} \cdot (\hat{\mathbf{z}} \times \mathbf{r}) + \frac{1}{2} m\Omega^2 (\hat{\mathbf{z}} \times \mathbf{r})^2 - m\frac{GM_S}{r_{SE}^2}\, \hat{\mathbf{r}}_{\mathrm{SE}} \cdot \mathbf{r} - \mathrm{m}U_{\mathrm{loc}} \tag{3.6}$$

where U_{loc} describes some local interaction, *e.g.*, between the planet and its moon. It is a matter of trivial exercise to show that these potentials can easily account for the popular "proofs" of Earth's rotation like the Foucault's Pendulum or the existence of the geostationary orbits.

4. Conclusion

We have presented the mathematical formalism which can justify Mach's statement that both geocentric and Copernican modes of view are "equally actual" and "equally correct."[26] This is performed by introducing two potentials: (1) a vector potential that accounts for the diurnal rotations and (2) a scalar potential that accounts for the annual revolutions of the celestial bodies

[26] Mach, E. 1960, *The Science of Mechanics*, 6th ed., LaSalle IL: Open Court, pp. xxviii, 279, 284.

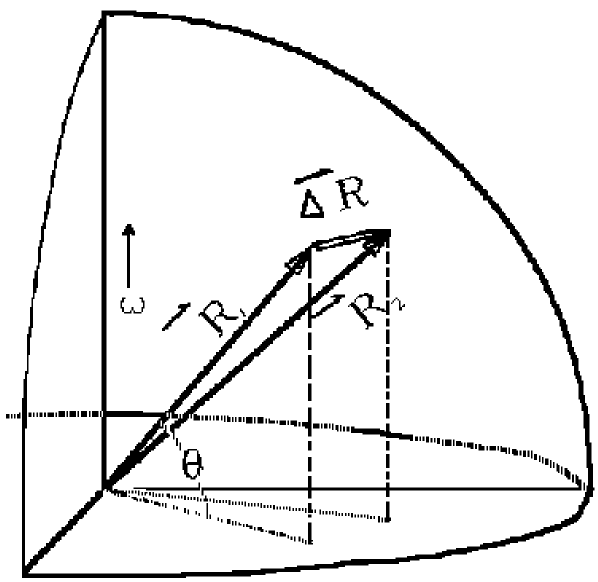
around the fixed Earth. These motions can be seen as real and self-sustained. If one could put the whole Universe in accelerated motion around the Earth, the potentials (3.1) and (3.5) would immediately be generated and would keep the Universe in that very same state of motion *ad infinitum*.

Additional Kinematical/Vector Analysis of a Rotating Universe

Now that we have analyzed geocentric mathematics by use of the dynamical approach using potential formalism, we will now demonstrate the mathematics by the kinematical approach, primarily using vectors. Our purpose is to show that the universe and its stars can revolve around a central point, Earth, once per day.

First, we will show how the National Weather Service calculates the Coriolis force. We will then use a similar means to calculate the inertial forces of a rotating universe around Earth.

The following diagram and analysis for deriving the Coriolis and centrifugal forces appears on the National Weather Service website.



The explanation from NWS is as follows:

> The derivation of the Coriolis force begins by simplifying the problem the most general problem, a transformation of a vector (the wind velocity) between a fixed and a rotating frame of reference the Earth rotating with an angular velocity.
>
> The derivation starts with this figure. Here R is the vector distance from the center of the Earth to the wind, and omega is the rotation vector of the Earth; it lies on the axis and points to the North Pole.
>
> Over a time period (Delta T) the distance Delta R is the velocity of the Earth plus the velocity of the wind relative to the Earth. The velocity of anything is defined as the first derivative of its position.

$$\vec{v} \equiv \frac{d\vec{R}}{dt} \qquad (1)$$

so the wind velocity as viewed from space is simply the wind velocity as viewed from Earth plus the velocity of Earth as viewed from space.

$$\frac{d\vec{R}}{dt_{Space}} = \frac{d\vec{R}}{dt_{Earth}} + \vec{\omega} \times \vec{R} \qquad (2)$$

Here, omega is the rotation rate of the Earth, once per day. To find the acceleration, we differentiate again following the normal rules of calculus like this,

$$\frac{d^2\vec{R}}{dt^2_{Space}} = \frac{d}{dt}\left(\frac{d\vec{R}}{dt_{Earth}}\right) +_{Space} \frac{d(\vec{\omega} \times \vec{R})}{dt} \qquad (3)$$

Doing the differentiation and collecting terms we arrive at

$$\frac{d^2\vec{R}}{dt^2_{Space}} = \frac{d^2\vec{R}}{dt^2_{Earth}} + 2(\vec{\omega} \times \vec{v})_{Coriolis} +$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{R})_{Centripetal} + \frac{d\vec{\omega}}{dt} \times \vec{R}_{Day\ length\ [Euler\ term]} \qquad (4)$$

There are three terms on the right which we must add when we are looking at the winds from the ground. The first is the Coriolis term, the second is the Centripetal term and the third is the effect of a changing rotation rate on the winds if the Earth's rotation rate changed. Fortunately, for studying the weather, only the Coriolis term is important, the other two can be neglected. So the problem reduces to

$$\vec{a}_{Space} = \vec{a}_{Earth} + 2(\vec{\omega} \times \vec{v})_{Coriolis\ "acceleration"} \qquad (5)$$

Since force is mass times acceleration, when calculating the forces from the ground, we add the Coriolis term to correct for our view from the rotating coordinate system.

$$\frac{\vec{F}}{m_{Space}} = \frac{\vec{F}}{m_{Earth}} + 2(\vec{\omega} \times \vec{v})_{Coriolis\ "Force"} \qquad (6)$$

Normally, we assume the mass of the fluid we are looking at is1 kilogram. If you want it out of vectors assuming you are looking at the horizontal component of the wind with respect to the Earth's turning surface, you have, for any latitude lambda,

$$\vec{F}_{Space} = \vec{F}_{Earth} + 2\,|\vec{\omega}|\,\sin(\lambda) \qquad (7)$$[27]

We will now take these principles and calculations and apply them to a rotating Universe around a fixed Earth. In order to demonstrate the rotation of the Universe we will use a single star since its coordinates are fixed to the sphere of the Universe which carries the star. A star's location is calculated by its ascension and declination. See Figure 1:

[27] http://www.nws.noaa.gov/om/wind/deriv.shtml

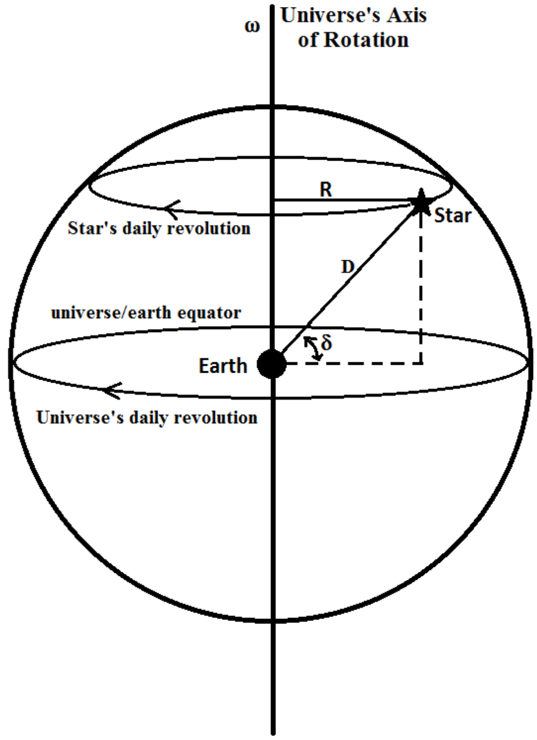


Figure 1

We will now derive the equations for the daily rotation of the Universe around a fixed Earth. The following terms will be used:

- F is the Universe's gravitational force exerted on the star
- a is the acceleration of the star as it revolves around the Earth
- R is the radius of the star's orbit around the Universe's axis
- v is the velocity of the star revolving around the Universe's axis
- D is the distance from the Earth to the star
- m is the star's mass
- δ is the star's declination as measured from the Earth's equator
- ω is the rotation rate of the Universe in degrees per second

Acceleration is the change in velocity over time, and is mathematically described as

$$\boldsymbol{a} = \frac{d^2\boldsymbol{R}}{dt^2} \tag{1}$$

where $\boldsymbol{R}$ is the distance to the star and t is the elapsed time the star has moved, which can also be expressed as

$$\boldsymbol{a} = \frac{d}{dt}\frac{d\boldsymbol{R}}{dt} \tag{2}$$

where $d\boldsymbol{R}/dt$ is the velocity ($\boldsymbol{v}$) of the star. Since the velocity of the star follows the equation

$$\boldsymbol{v} = \frac{dR}{dt} = -\boldsymbol{\omega} \times \boldsymbol{R} \tag{3}$$

where ω is the angular velocity in degrees per second and $\boldsymbol{R}$ is the distance of the star from the Universe's axis of rotation, thus we must rewrite equation 2 as

$$a = \frac{d}{dt}(-\boldsymbol{\omega} \times \boldsymbol{R}) \tag{4}$$

Now we use the calculus differentiation of the derivative $\frac{d}{dt}$ through all the terms and arrive at

$$a = -\frac{d\omega}{dt} \times \boldsymbol{R} - 2\boldsymbol{\omega} \times \boldsymbol{v} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{R}) \tag{5}$$

As in the National Weather Service equations,

- $2\boldsymbol{\omega} \times \boldsymbol{v}$ is the Coriolis force
- $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{R})$ is the centrifugal force
- $\frac{d\omega}{dt} \times \boldsymbol{R}$ is the Euler force (which doesn't apply since there is no variation in the length of day)

In the geocentric system, the centrifugal and Coriolis forces are the main forces that move the sun, stars and planets, and thus we will eliminate the Euler force. This reduces equation 5 to:

$$\boldsymbol{a} = -2\boldsymbol{\omega} \times \boldsymbol{v} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{R}) \tag{6}$$

where v is the velocity of the star revolving around the Universe's axis. Since the Universe is rotating as opposed to the Earth, the velocity is in the opposite direction of the heliocentric system, and thus the velocity is given a negative sign.

Since the velocity (v) in equation 6 is $-(\omega \times \boldsymbol{R})$, equation 6 becomes

$$\boldsymbol{a} = 2\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{R}) - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{R}) \tag{7}$$

which is simplified to

$$\boldsymbol{a} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{R}) \tag{8}$$

Distributing the cross-product through the terms yields

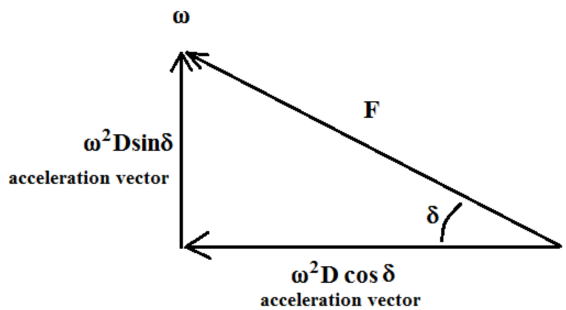
$$\boldsymbol{a} = \boldsymbol{\omega}(\boldsymbol{\omega} \cdot \boldsymbol{R}) - \boldsymbol{R}(\boldsymbol{\omega} \cdot \boldsymbol{\omega}) \tag{9}$$

Since the star is located at declination δ, then (ω × $\boldsymbol{R}$) equals $D\omega$ sin(δ), which yields

$$\boldsymbol{a} = -\omega^2(\boldsymbol{R} - D\widehat{\omega}\sin(\delta)) \tag{10}$$

where $\widehat{\omega}$ is a vector of the rotation axis in the direction of ω, which is in the plane of the star's orbit, as in Figure 1. The $\widehat{\omega}$ maintains the star's acceleration at the star's presumed declination that is determined by $\boldsymbol{R}$.

Equation 10 has two acceleration vectors which are depicted in Figure 2. After multiplying both vectors by the mass of the star, the sine term ($\omega^2 D\sin\delta$) shows what keeps the star's plane of rotation stable in the vertical frame (*i.e.*, not going up or down), while the cosine term ($\omega^2 D\cos\delta$) shows what pulls the star toward the Universe's axis of rotation. The net result of the two vector accelerations is to maintain the star in its position within the inertial field created by the gravitation of the Universe.



Since Equation 10 is kinematic (dealing with motion) as opposed to dynamic (dealing with forces), we will make the equation dynamic in order to show that the geocentric universe is dynamically consistent. Before we do so, it is necessary to show how modern physicists do the same with common kinematic equations. For example, the velocity of a body in circular motion is understood as:

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{R}$$

But this is a kinematic equation, since it only describes the motion, not the forces causing the motion. To make it dynamic we can multiply both sides by the mass (*m*) to get:

$$m\boldsymbol{v} = m\boldsymbol{\omega} \times \boldsymbol{R}$$

In this case, modern physics will replace *mv* with ***p***, which is called the "momentum," so that:

$$\boldsymbol{p} = m\boldsymbol{\omega} \times \boldsymbol{R}$$

We will now do the same procedure for producing the dynamic version of the geocentric universe. To do so, we will multiply each side of Equation 10 [$\boldsymbol{a} = -\omega^2(\boldsymbol{R} - D\widehat{\omega}\sin(\delta))$] by the star's mass (*m*), to get:

$$m\boldsymbol{a} = -m\omega^2(\boldsymbol{R} - D\widehat{\omega}\sin(\delta)) \qquad (11)$$

and since $ma = F$, then the Force which keeps each celestial object in its designated place in a rotating Universe is:

$$\boldsymbol{F} = -m\omega^2(\boldsymbol{R} - D\widehat{\omega}\sin(\delta)) \qquad (12)$$

Not only will each celestial object be held in place by this equation, light itself will also obey this equation. In the geocentric Universe light can assume any speed since (a) limitations to light speed do not apply to rotating frames, and (b) inertial forces can accelerate or decelerate light's speed.[28]

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[28] My thanks to Dr. Gerry Bouw for his help in arriving at this vector analysis and his permission to use it.

The Geocentric Lagrange Points[29]

In the heliocentric system, we have two bodies with mass M_1 and M_2 and an angular velocity Ω, with the distance between M_1 and M_2 equal to R.

Using Kepler's law, we have the relationship between the angular velocity, the radius R and the masses of the bodies:

$$\Omega^2 R^3 = G(M_1 + M_2)$$

We note here that the heliocentric Lagrange points are at a distance "*R*" from the sun to the Earth, not to the center of mass between the sun and Earth.

In the geocentric system, we have the same angular velocity Ω of the sun relative to the Earth.

As the heliocentric Lagrange points are at a distance *R* from the Sun to the Earth, the same is true in the geocentric model.

Before we show these Lagrange points, let's first establish that, in the heliocentric system, the effective force in a frame that is rotating with angular velocity Ω is related to the inertial force $\vec{F}$, which is noted in the following transformation:

$$\vec{F}_\Omega = \vec{F} - 2m\left(\vec{\Omega} \times \frac{d\vec{r}}{dt}\right) - m\vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right) \quad (1)$$

The first part $[2m\left(\vec{\Omega} \times \frac{d\vec{r}}{dt}\right)]$ is the Coriolis force and the second $[m\vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right)]$ is the centrifugal force.

The effective force can be derived from the generalised potential:

$$U_\Omega = U - \vec{v}\left(\vec{\Omega} \times \vec{r}\right) + ½\left(\vec{\Omega} \times \vec{r}\right)\left(\vec{\Omega} \times \vec{r}\right) \quad (2)$$

which can be related to the generalized gradient:

$$\vec{F}_\Omega = -\nabla_{\vec{r}} U_\Omega + \frac{d}{dt}\left(\nabla_{\vec{r}} U_\Omega\right) \quad (3)$$

The graph of generalized potential for specific values is:

$$\vec{v} = 0, M_1 = 10, M_2 = 1, R = 10 \quad (4)$$

The role of these coefficients is to obtain Ω, which is the same in both the heliocentric and the geocentric systems.

In the geocentric, the angular velocity Ω proceeds from the initial conditions of the system, which is the rotating universe.

The generalized potential uses the same values for Ω and R as in heliocentrism. The difference is that we use a non-central potential[30], but the expression for the generalized potential is the same, and its graph and its extreme regions L1 ... L5 will be the same. Note the following:

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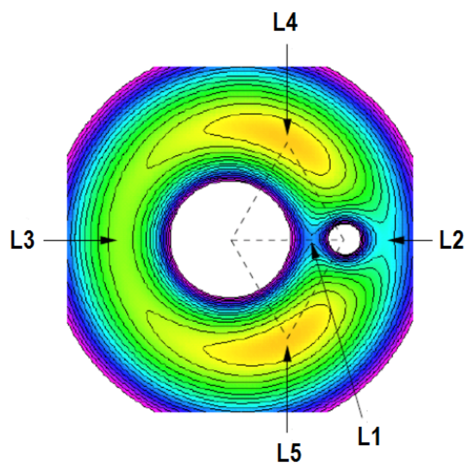
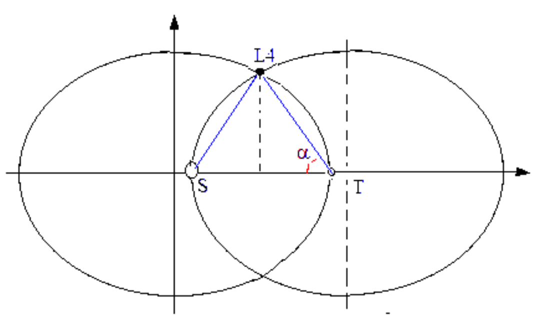
[29] My thanks to Dr. Milenko Bernadic, Ph.D. in mathematics for his help on the Lagrange calculations.



We can then arrive at Lagrange points L4 and L5 based on the dynamic equivalence of the two models. First, L4 and L5 must be in the intersection of the two paths. Therefore, we do not need to consider circular paths as in the general problem of two points, but can start from the elliptical paths, or even observable paths without using a geometric model, to determine their intersection. In this case, placing the sun and Earth in the respective focus of the ellipses, we have:



Let us suppose the center of the ellipse is the origin of the Cartesian coordinates. The triangle STL is an isosceles triangle and we now calculate the value of the angle α.

We have: $a + c = a\,(1 + \varepsilon) = aI$. Where we call $I = 1 + \varepsilon$.

We then calculate the coordinates of L4 in the Cartesian equations of the two ellipses:

$$\begin{pmatrix} \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \\ \left(\frac{x - aI}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \end{pmatrix} \rightarrow x^2 = (x - aI)^2 \; i.e., x = \frac{1}{2}\,aI \tag{5}$$

$$\left(\frac{y}{b}\right)^2 = 1 - \left(\frac{½\,aI - aI}{a}\right)^2 \rightarrow y = b\,\left(1 - \frac{1}{4}\,I^2\right)^{½} \tag{6}$$

Since $I = 1 + \varepsilon$ with $\varepsilon \rightarrow 0$, we have:

$$y \approx b\,\left(1 - \frac{1}{4} - \frac{\varepsilon}{2}\right)^{½} \approx \frac{\sqrt{3}}{2}\,b \tag{7}$$

For the sun-earth system we have:

[30] As developed by Dr. Luka Popov earlier in this chapter.

a = 1,000000, b = 0,999863, ε = 0,016711.

Thus:

$$\alpha = \arctan\left(\frac{\sqrt{3}b}{a(1+\varepsilon)}\right) = 59°35'1.3'' \qquad (8)$$

and thus locates the L4 point.

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