How Can the Larger Sun Revolve around the Smaller Earth?

This question is one of the most frequently asked in regard to a geocentric universe. Since the sun has 99% of the mass of the solar system, it seems counterintuitive that it could revolve around the tiny Earth, especially when we see smaller moons revolving around larger planets.

This objection is not without merit. If our world were confined to a sun, Earth and planets, it would certainly be the case that the smaller planets, including Earth, would revolve around the sun. This is precisely why Newton believed that the Earth revolves around the sun. He limited his physics to our solar system.

But Newton was smart enough to realize that if he expanded his system to include the forces in the rest of the universe, he agreed that the Tychonic geocentric system would be viable. Here is what he said in Proposition 43:

> In order for the Earth to be at rest in the center of the system of the Sun, Planets, and Comets, there is required both universal gravity and another force in addition that acts on all bodies equally according to the quantity of matter in each of them and is equal and opposite to the accelerative gravity with which the Earth tends to the Sun…
>
> Since this force is equal and opposite to its gravity toward the Sun, the Earth can truly remain in equilibrium between these two forces and be at rest. And thus celestial bodies can move around the Earth at rest, as in the Tychonic system.

Notice that Newton specifies that there must be "another force…that acts on all bodies equally." Well, that "force" appears when the universe is allowed to rotate around the Earth. That universal "force" acts upon every celestial body and keeps them in their daily rotation around the fixed Earth.

This can be proven by using the mathematics of both Newton's physics with Mach's physics and applying them to a rotating universe. The mathematics will be demonstrated by the attached paper by my good friend, Dr. Luka Popov from Croatia. A few years ago, I had asked Dr. Luka to write the paper and he graciously accepted. The paper turned out to be so good that Dr. Popov sought to have it published. Soon thereafter it was published in the *European Journal of Physics* in January 2013, even though that magazine has no partiality toward geocentrism and is considered a mainstream publication that fully supports Copernicanism, Relativity and Darwinism. In fact, Dr. Popov's paper was almost published in the *American Journal of Physics*, except that it lost out by one vote, probably because the referees did not prefer the geocentrism the paper was making viable.

Incidentally, Newton's allowance of geocentrism in Proposition 43 was recently noticed by a famous American physicist, Stephen Weinberg. Here is what he said in his recent book, *To Explain the World*:

> If we were to adopt a frame of reference like Tycho's in which the Earth is at rest, then the distant galaxies would seem to be executing circular turns once a year, and in general relativity this enormous motion would create forces akin to gravitation, which would act on the Sun and planets and give them the motions of the Tychonic theory. Newton seems to have had a hint of this. In an unpublished 'Proposition 43' that did not make it into the *Principia*, Newton acknowledges that Tycho's theory could be true if some other force besides ordinary gravitation acted on the Sun and planets.[1]

Notice that Weinberg says that if the universe rotated around a fixed Earth, "in general relativity this enormous motion would create forces akin to gravitation, which would act on the Sun and planets and give them the motions of the Tychonic theory." In other words, it wasn't only Newton who recognized that the force created by a rotating universe would keep the sun revolving around the Earth, but Albert Einstein's theory of General Relativity does the same. Weinberg is just making note of the fact that Newton said it before Einstein.

So here we have, the two greatest systems of physics developed by mankind both saying that a geocentric system is viable. How much more evidence do we need?

Our illustrious critics, Alex MacAndrew and David Palm had castigated us for using Einstein's allowance of geocentrism because, they claimed, "they can't use a theory that they don't believe in," even though they themselves believe in Einstein and thus would have to concede, from their own views, that geocentrism is viable.

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[1] Steven Weinberg, *To Explain the World: The Discovery of Modern Science*, HarperCollins, 2015, pp. 251-252.

But now we have evidence from Isaac Newton himself that if we apply Newtonian mechanics to a rotating universe, we have another confirmation of geocentrism.

Hence, the game is over and it is just a matter of calling out the band to celebrate the victory. My guess is that Messers MacAndrew and Palm will pretend that all is well and make up some excuse why Newton's physics is not applicable, just as they tried to do with Einstein.

With that warning, I turn you over to Dr. Popov and his paper. Pay special attention to Section 3 near the end, since it will give the equations for both an annual and diurnal revolution of the sun around the Earth, and show why the sun will remain in lockstep as it moves with the universe.

Robert Sungenis
February 19, 2016

A Newtonian-Machian Mathematical Analysis of Neo-tychonian Model of Planetary Motions[2]

The calculation of the trajectories in the Sun-Earth-Mars system will be performed in two different models, both in the framework of Newtonian mechanics. First model is the well-known Copernican system, which assumes the Sun is at rest and all the planets orbit around it. Second one is less known model developed by Tycho Brahe (1546-1601), according to which the Earth stands still, the Sun orbits around the Earth, and other planets orbit around the Sun. The term "Neo-tychonian system" refers to the assumption that orbits of distant masses around the Earth are synchronized with the Sun's orbit. It is the aim of this paper to show the kinematical and dynamical equivalence of these systems, under the assumption of Mach's principle.

The discussion of motion of celestial bodies is one of the most interesting episodes in the history of science. There are two diametrically opposite schools of thought: one that assumes that the Sun stands still, and Earth and other planets orbit around it; and another that assumes that the Earth stands still, and Sun and other planets in some manner orbit around the Earth. The first school of thought comes from Aristarchus (310-230 BC) and is generally addressed as heliocentrism, another from Ptolemy (90-168 BC) and is generally known as geocentrism. Since Aristotle, the ultimate authority in science for more than two millennia, accepted the geocentric assumption, it became dominant viewpoint among scientists of the time. The turnover came with Copernicus (so-called "Copernican revolution") who in his work *De Revolutionibus* proposed a hypothesis that the Sun stands in the middle of the known Universe, and that Earth orbits around it, together with other planets. Copernicus' system was merely better than Ptolemy's, because Copernicus assumed the trajectories of the planets are perfect circles, and required the same number of epicycles (sometimes even more) as Ptolemy's model.[3] The accuracy of Ptolemy's model is still a subject of vivid debates among historians of science.[4]

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[2] This paper was accepted for publication by the *European Journal of Physics* in January 2013. L. Popov, "Newtonian–Machian analysis of the neo-Tychonian model of planetary motions," Eur. J. Phys. **34**, 383-391 (2013). The author is Luka Popov. Also available at arXiv:1301.6045 [physics.class-ph].

[3] Koestler, A., 1959 *The Sleepwalkers: A History of Man's Changing Vision of the Universe,* London: Hutchinson, pp. 194–5.

[4] Rawlins, D., 1987, "Ancient heliocentrists, Ptolemy, and the equant" *Am. J. Phys*. 55 235–9.

The next episode in this controversy is Kepler's system with elliptical orbits of planets around the Sun. That system did not require epicycles, it was precise and elegant. It is therefore general view that Kepler's work finally settled the question whether it is the Sun or the Earth that moves. But what is less known is that Tycho Brahe, Kepler's tutor, developed a geostatic system that was just as accurate and elegant as Kepler's: the Sun orbits around the Earth, and all the other planets orbit around the Sun. The trajectories are ellipses, and all the Kepler's laws are satisfied. In that moment of history, the Kepler's and Brahe's models were completely equivalent and equally elegant, since neither of them could explain the mechanism and reason why the orbits are the way they are. It had to wait for Newton.

Sir Isaac Newton, as it is generally considered, gave ultimate explanation of planetary motions that was in accord with Kepler's model, and excluded Brahe's one. The laws of motions and the inverse square law of gravity could reproduce all the observed data only with the assumption that the Sun (*i.e.* the center of mass of the system, which can be very well approximated by the center of the Sun) stands still, and all planets move around it. According to Newton's laws, it is impossible for the small Earth to keep the big Sun in its orbit: the gravitational pull is just too weak. This argument is very strong, and it seems to settle the question for good.

But in the end of 19th century, the famous physicist and philosopher Ernst Mach (1839-1916) came with the principle which states the equivalence of non-inertial frames. Using the famous "Newton's bucket" argument, Mach argues that all so-called pseudo-forces (forces which results from accelerated motion of the reference frame) are in fact real forces originating from the accelerated motion of distant masses in the Universe, as observed by the observer in the non-inertial frame. Some even go further, stating that "every single physical property and behavioral aspect of isolated systems is determined by the whole Universe."[5] According to Mach's principle, the Earth could be considered as the "pivot point" of the Universe: the fact that the Universe is orbiting around the Earth will create the exact same forces that we usually ascribe to the motion of the Earth.

Mach's principle played a major role in the development of Einstein's general theory of relativity,[6] as well as other developments in gravitational theory, and has inspired some interesting experiments.[7] This principle still

[5] Rosen, J., 1981, "Extended Mach principle," *Am. J. Phys*., 49, 258–64.
[6] Newburgh, R., 2007, "Inertial forces, absolute space, and Mach's principle: the genesis of relativity," *Am. J. Phys*., 75, 427–30;
[7] Lichtenegger, H. and Mashhoon, B., 2004, "Mach's principle," arXiv:physics/0407078 [physics.hist-ph]

serves as a guide for some physicists who attempt to reformulate ('Machianize') Newtonian dynamics[8] [9], or try to construct new theories of mechanics.[10] Some arguments against and critiques of Mach's principle have also been raised.[11] Since the time of its original appearance[12], Mach's principle has been reformulated in a number of different ways[13]. For the purpose of this paper, we will focus on only one of the consequences of Mach's principle: that the inertial forces can be seen as resulting from real interactions with distant matter in the Universe, as was for example shown by Zylbersztajn.[14]

The only question that remains is: *are these forces by themselves enough to explain all translational motions that we observe from Earth, and can they reproduce the Tycho Brahe's model*? The discussion in this paper will show that the answer to this question is positive. In order to demonstrate it, we will consider the Sun-Earth-Mars system.

The paper is organized as follows. In section 2 an overview of two-body problem in the central potential and Kepler's problem is given. In section 3 the calculations of Earth's and Mars' trajectories are performed in the heliocentric system, both analytically (by applying the results from previous section) and numerically. In section 4 the calculations of Sun's and Mars' trajectories are performed in geocentric system, due to the presence of pseudo-potential originating from the fact of accelerated motion of the Universe. Finally, the conclusion of the analysis is given.

2. Two-Body Problem in the Central Potential

2.1 General overview

We start with the overview of two body problem in Newtonian mechanics. Although there are alternative and simpler ways to solve this

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[8] Hood, C. G., 1970, "A reformulation of Newtonian dynamics," *American Journal of Physics.*, 38, 438–42.
[9] Barbour, J., 1974, "Relative-distance Machian theories," *Nature*, 249, 328.
[10] Assis, A. K. T., 1999, *Relational Mechanics*, Montreal: Aperion.
[11] Hartman, H. I. and Nissim-Sabat, C., 2003, "On Mach's critique of Newton and Copernicus," *Am. J. Phys.*, 71, 1163–8.
[12] Mach, E. 1872, *Die Geschichte und die Wurzel des Satzes von der Erhaltung der Arbeit*, Prague: Calve; Mach, E., 1883, *Die Mechanik in ihrer Entwickelung Historisch–Kritisch Dargestellt*, Leipzig: Brockhaus; Mach, E., 1911, *History and Root of the Principle of the Conservation of Energy*, Chicago, IL: Open Court.
[13] Rovelli, C., 2004, *Quantum Gravity*, Cambridge: Cambridge University Press, p. 75; Barbour, J., 2010, "The definition of Mach's principle," arXiv:1007.3368.
[14] Zylbersztajn, A., 1994, "Newton's absolute space, Mach's principle and the possible reality of fictitious forces," *Eur. J. Phys.*, 15, 1–8.

problem,[15] we will follow the usual textbook approach.[16] The Lagrangian of the system reads:

$$L = \tfrac{1}{2}m_1\dot{\mathbf{r}}_1^2 + \tfrac{1}{2}m_2\dot{\mathbf{r}}_2^2 - U(|\mathbf{r}_1 - \mathbf{r}_2|) \tag{2.1}$$

where U is potential energy that depends only on the magnitude of the difference of radii vectors (so-called central potential). We can easily rewrite this equation in terms of relative position vector $\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$, and let the origin be at the center of mass, *i.e.*, $m_1\mathbf{r}_1 + m_2\mathbf{r}_2 \equiv 0$. The solution of these equations is:

$$\mathbf{r}_1 = \frac{m_2}{m_1 + m_2}\mathbf{r}, \qquad \mathbf{r}_2 = -\frac{m_2}{m_1 + m_2}\mathbf{r}\,\cdot \tag{2.2}$$

The Lagrangian (2.1) so becomes

$$\mathrm{L} = \tfrac{1}{2}\mu\dot{\mathbf{r}}^2 - U(r) \tag{2.3}$$

where $\mathbf{r} \equiv |\mathbf{r}|$ and μ is the reduced mass,

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} \tag{2.4}$$

In that manner, the two-body problem is reduced to one body problem of particle with coordinate $\mathbf{r}$ and mass μ in the potential $U(r)$.

Using polar coordinates, the Lagrangian (3) can be written as:

$$\mathrm{L} = \tfrac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2) - U(\mathrm{r}) \tag{2.5}$$

One can immediately notice that variable ϕ is cyclic (it does not appear in the Lagrangian explicitly). Consequence of that fact is momentum conservation law, since $(\partial/\partial t)\,(\partial L/\partial\dot{\phi}) = \partial L/\partial\phi = 0$. Therefore,

$$\ell \equiv \frac{\partial L}{\partial\dot{\phi}} = \mu r^2\dot{\phi} = \text{const.} \tag{2.6}$$

is the integral of motion.

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[15] Hauser, W., 1985, "On planetary motion," *Am. J. Phys*., 53, 905–7; Gauthier, N., 1986, "Planetary orbits," *Am. J. Phys*., 54, 203.

[16] Landau, L. D. and Lifshiz, E. M., 1976, *Mechanics*, 3rd edn., Oxford: Butterworth-Heinemann, pp. 25–40; Goldstein, H., 1980, *Classical Mechanics*, 2nd edn., Reading, MA: Addison-Wesley, pp. 70–102.

In order to find a solution for the trajectory of a particle, it is not necessary to explicitly write down the Euler-Lagrange equations. Instead, one can use the energy conservation law,

$$E = ½\mu(\dot{r}^2 + r^2\dot{\phi}^2) + U(r) = ½\mu\dot{r}^2 + \frac{\ell^2}{2\mu r^2} + U(r) \tag{2.7}$$

Straightforward integration of (2.7) gives the equation for the trajectory,

$$\phi(r) = \int \frac{\ell \, \mathrm{d}r/r^2}{\sqrt{2m[E - U(r) - \ell^2/r^2}} \tag{2.8}$$

2.2 Kepler's problem

Let us now consider the particle in the potential

$$U(r) = -\frac{k}{r} \tag{2.9}$$

generally known as *Kepler's problem*. Since our primary interest is in the planetary motions under the influence of gravity, we will take $k > 0$. Solution of eq. (8) for that potential is:

$$\frac{p}{r} = 1 + e \cos \phi, \tag{2.10}$$

where $2p$ is called the *lactus rectum* of the orbit, and e is the eccentricity. These quantities are given by

$$p = \frac{2\ell^2}{\mu k}, \qquad e = \sqrt{1 + \frac{2E\ell^2}{\mu k^2}} \tag{2.11}$$

Expression (2.10) is the equation of a conic section with one focus in the origin. For $E < 0$ and $e < 1$ the orbit is an ellipse.

One can also determine minimal and maximal distances from the source of the potential, called perihelion and aphelion, respectively:

$$r_{min} = \frac{p}{1+e}, \qquad r_{max} = \frac{p}{1-e} \tag{2.12}$$

These parameters can be directly observed, and often are used to test a model or a theory regarding planetary motions.

3. Earth and Mars in the Heliocentric Perspective

According to Newton's law of gravity, the force between two massive objects reads:

$$\mathbf{F} = \frac{Gm_1m_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}(\mathbf{r}_1 - \mathbf{r}_2) \tag{3.1}$$

Which leads to a potential ($\mathbf{F} = -\nabla U$)

$$U(|\mathbf{r}_1 - \mathbf{r}_2|) = -\frac{Gm_1m_2}{|\mathbf{r}_1 - \mathbf{r}_2|} \tag{3.2}$$

This is obviously Kepler's potential (2.9) with $k = Gm_1m_2$, where G is Newton's gravitational constant.

Since the Sun is more than 5 orders of magnitude more massive than Earth and Mars, we will in all future analysis use the approximation

$$\mu \approx m_i \tag{3.3}$$

where m_i is mass of the observed planet. For the same reason, gravitational interaction between Earth and Mars can be neglected, since it is negligible compared with the interaction between Mars and the Sun. Using these assumptions, we can write down corresponding Lagrangians,

$$L_{ES} = \tfrac{1}{2}m_E\dot{\mathbf{r}}_{ES}^2 + \frac{Gm_EM_S}{\mathbf{r}_{ES}},$$

$$L_{MS} = \tfrac{1}{2}m_M\dot{\mathbf{r}}_{MS}^2 + \frac{Gm_MM_S}{\mathbf{r}_{MS}} \tag{3.4}$$

where m_E and m_M are masses of Earth and Mars, respectively. Subscripts *ES* (*MS*) correspond to the motion of Earth (Mars) with respect to the Sun. These trajectories can be calculated using the exact solution (2.10) with appropriate strength constants k and initial conditions which determine E and ℓ. Another way is to solve the Euler-Lagrange equations numerically, using astronomical parameters[17] (*e.g.*, aphelion and perihelion of Earth/Mars) to choose the initial conditions that fit the observed data. The

[17] Weast, R. C. (ed), 1968, *Handbook of Chemistry and Physics*, 49th edn., Cleveland, OH: Chemical Rubber Company, pp. F145–6.

former has been done using *Wolfram Mathematica* package. The result is shown on Fig. 1.

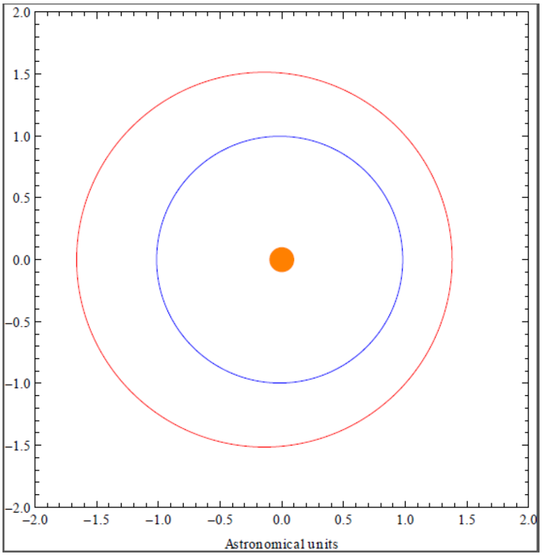

FIG. 1: Trajectories of Earth and Mars in heliocentric system over the period of 2 years. Blue and red lines represent Earth's and Mars' orbits, respectively.

For the later comparison, one could write out the expressions for the *e* and *p* parameters for the Earth. Putting the expressions for energy (2.7) and momentum (2.6) into eqs. (2.11) it is straightforward to obtain

$$p = \frac{\dot{\phi}^2 r^4}{G M_S}$$

$$e = \sqrt{1 - \frac{2 G M_S \dot{\phi}^2 r^3 - \dot{r}^2 \dot{\phi}^2 r^4 - \dot{\phi}^4 r^6}{G^2 M_S^2}} \tag{3.5}$$

where $\dot{\phi}$, $\dot{r}$ and r are angular velocity, radial velocity and distance respectively, taken in the same moment of time (*e.g.* in $t = 0$).

Fig. 2 displays motion of the Mars as viewed from the Earth, gained by trivial coordinate transformation

$$\mathbf{r}_{EM}(t) = -\mathbf{r}_{ES}(t) + \mathbf{r}_{MS}(t), \tag{3.6}$$

where $\mathbf{r}_{ES}(t)$ and $\mathbf{r}_{MS}(t)$ are solutions of Euler-Lagrange equations for the Lagrangians (3.4). Equation (3.6) is just the mathematical expression of the Tycho Brahe's claim. The retrograde motion of Mars can be useful in the attempt to understand and determine orbital parameters, as was shown qualitatively and quantitatively by Thompson.[18]

The acceleration that Earth experiences due to the gravitational force of the Sun is usually referred as centripetal acceleration and is given by

$$\mathbf{a}_{cp} = \frac{\mathbf{F}_{cp}}{m_E} = \frac{GM_S}{r_{ES}^2}\,\hat{\mathbf{r}}_{\mathrm{ES}} \tag{3.7}$$

where $\hat{\mathbf{r}}$ is the unit vector in the direction of vector $\mathbf{r}$, $\mathbf{r}_{\mathrm{ES}}(t)$ is radius vector describing motion of Earth around the Sun, and F_{cp} is centripetal force, *i.e.* the force that causes the motion.

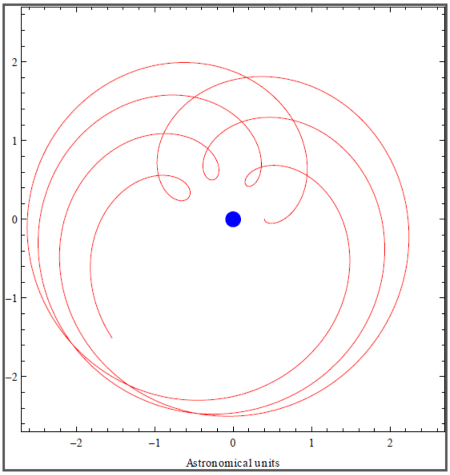

FIG. 2: Trajectory of the Mars as seen from the Earth over the period of 7 years. Calculation of this trajectory is done numerically in the heliocentric system.

[18] Thompson, B. G. 2005, "Using retrograde motion to understand and determine orbital parameters," *Am. J. Phys.*, 73, 1023–9.

4. Sun and Mars in the Geocentric Perspective

4.1 The pseudo-potential

From the heliocentric perspective, the fact that the Earth moves around the Sun results with centrifugal pseudo-force, observed only by the observer on the Earth. But if we apply Mach's principle to the geocentric viewpoint, one is obliged to speak about the real forces resulting from the fact that the Universe as a whole moves around the observer on the stationary Earth. Although these forces will further be considered as the real forces, we well keep the usual terminology and call them pseudo-forces, for the sake of convenience. Our focus here will be on the annual orbits, not on diurnal rotation which requires some additional physical assumptions[19] that are beyond the scope of this paper.

The Universe is regarded as an $(N + 1)$ –particle system (N celestial bodies plus planet Earth). From the point of a stationary Earth, one can write down the Lagrangian that describes the motions of celestial bodies:

$$L = ½\sum_{i=1}^{N} m_i\, \dot{\mathrm{r}}_i^2 - ½\sum_{i=1}^{N} \frac{Gm_i m_j}{r_{ij}} - \sum_{i=1}^{N} \frac{Gm_E m_i}{r_i} - U_{ps}, \tag{4.1}$$

where $r_{ij} \equiv |\mathbf{r}_i - \mathbf{r}_j|$, U_{ps} stands for the pseudo-potential, satisfying $\mathbf{F}_{ps} = -\nabla U_{ps}$. $\mathbf{F}_{ps}$ is the pseudo-force given by

$$\mathbf{F}_{ps} = -m\sum_{i=1}^{N} \mathbf{a}_{cp,i}\,, \tag{4.2}$$

where $\mathbf{a}_{\mathrm{cp,i}}$ is centripetal acceleration for given celestial body (with respect to the Earth) and m is a mass of the object that is subjected to this force. It's easy to notice that the dominant contribution in these sums comes from the Sun. The close objects (planets, moons, etc.) are much less massive than the Sun, and massive objects are much farther distant. The same approximation is implicitly used in section 3.

In the Machian picture, the centripetal acceleration is a mere relative quantity, describing the rate of change of relative velocity. Therefore, centripetal acceleration of the Sun with respect to Earth is given by Equation 3.7, with $\mathbf{r}_{ES} = -\mathbf{r}_{SE}$. All that considered, Equation 4.2 becomes

[19] Vetö, B., 2011, "Gravitomagnetic field of the universe and Coriolis force on the rotating Earth," *Eur. J. Phys.*, 32, 1323–9; Assis, A. K. T., 1999, *Relational Mechanics*, Montreal: Aperion.

$$\mathbf{F}_{ps} = \frac{GmM_S}{r_{SE}^2}\,\hat{\mathbf{r}}_{SE} \tag{4.3}$$

where $\mathbf{r}_{SE}(t)$ describes the motion of the Sun around the Earth.

We can now finally write down the pseudo-potential which influences every body observed by the fixed observer on Earth:

$$U_{ps}(\mathbf{r}) = \frac{Gm\,M_S}{r_{SE}^2}\,\hat{\mathbf{r}}_{\mathrm{SE}} \cdot \mathbf{r} \tag{4.4}$$

where $\mathbf{r}(t)$ describes motion of particle of mass m with respect to the Earth. Notice that this is not a central potential.

4.2 The Sun in Earth's pseudo-potential

In order to determine Sun's orbit in Earth's pseudo-potential, one needs to take the dominant contributions of the Lagrangian (4.1), as was explained earlier. Taking into account the expression for pseudo-potential given in Equation 4.4, one ends up with

$$L_{SE} = \tfrac{1}{2} M_S \dot{\mathrm{r}}_{SE}^2 - \frac{GM_S^2}{r_{SE}} \tag{4.5}$$

This Lagrangian has the exact same form as the reduced Lagrangian (2.3). That means that we can immediately determine the orbit by means of Equation (2.11) by substituting $\mu = M_S$ and $k = GM_S^2$. This leads to the following result (subscript SE will be omitted):

$$p = \frac{\dot{\phi}^2 r^4}{GM_S}$$

$$e = \sqrt{1 - \frac{2GM_S\dot{\phi}^2 r^3 - \dot{r}^2\dot{\phi}^2 r^4 - \dot{\phi}^4 r^6}{G^2 M_S^2}} \tag{4.6}$$

which is the exact equivalent of the previous result given in Equations (3.5), since $\dot{\phi}$, $\dot{r}$ and r are relative quantities, by definition equivalent in both models. We can therefore conclude that the Sun's orbit in the Earth's pseudo-potential is equivalent as one observed from the Earth in the heliocentric system. It remains to show the same thing for Mars' orbit.

4.3 Mars in Earth's pseudo-potential

In the similar way as before, we take the dominant contributions of Lagrangian (4.1) together with Equation (4.4) and form the Lagrangian:

$$L_{ME} = \tfrac{1}{2} m_M \dot{\mathbf{r}}_{ME}^2 + \frac{G m_M M_S}{|\mathbf{r}_{ME} - \mathbf{r}_{SE}|} - \frac{G m_M M_S}{r_{SE}^2} \hat{\boldsymbol{r}}_{SE} \cdot \boldsymbol{r}_{ME} \tag{4.7}$$

where subscript *ME* refers to the motion of Mars with respect to Earth, and $\mathbf{r}_{SE}(t)$ is solution of the Euler-Lagrange equations for the Lagrangian (4.5).

The Euler-Lagrange equations for $\boldsymbol{r}_{ME}(t)$ Lagrangian (4.7) are too complicated to be solved analytically, but can easily be solved numerically. The numerical solutions for equations of motion for both the Sun and Mars are displayed in Fig. 3. The equivalence of trajectories gained in two different ways is obvious, justifying the model proposed by Tycho Brahe.

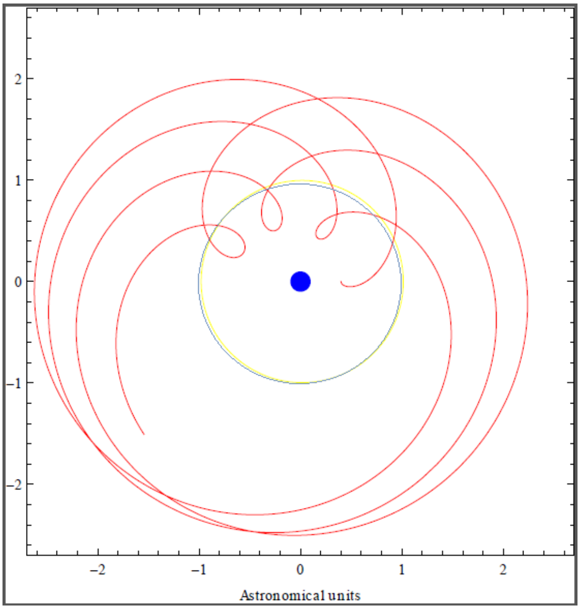

FIG. 3: Trajectories of the Sun (dark, blue) and the Mars (light, red) moving in Earth's pseudo-potential over the period of 7 years.

Calculation of this trajectory is performed numerically in the geocentric system.

5. Conclusion

The analysis of planetary motions has been performed in the Newtonian framework with the assumption of Mach's principle. The kinematical equivalence of the Copernican (heliocentric) and the Neo-tychonian (geocentric) systems is shown to be a consequence of the presence of a pseudo-potential (4.4) in the geocentric system, which, according to Mach, must be regarded as the real potential originating from the fact of the simultaneous acceleration of the Universe. This analysis can be done on any other celestial body observed from the Earth. Since Sun and Mars are chosen arbitrarily, and there is nothing special about Mars, one can expect to come up with the same general conclusion.

There is another interesting remark that follows from this analysis. If one could put the whole Universe in accelerated motion around the Earth, the pseudo-potential corresponding to the pseudo-force (4.2) will immediately be generated. That same pseudo-potential then causes the Universe to stay in that very state of motion, without any need of exterior forces acting on it. See the following.

Using Mach's principle, we will show that the observed diurnal and annual motion of the Earth can just as well be accounted as the diurnal rotation and annual revolution of the Universe around the fixed and centered Earth. This can be performed by postulating the existence of vector and scalar potentials caused by the simultaneous motion of the masses in the universe, including the distant stars.

1. Introduction

The modern day use of the word relativity in physics is usually connected with Galilean and special relativity, *i.e.*, the equivalence of the systems performing the uniform rectilinear motion, so-called inertial frames. Nevertheless, the physicists and philosophers never ceased to debate the various topics under the heading of Mach's principle, which essentially claims the equivalence of all co-moving frames, including non-inertial frames as well.

Historically, this issue was first brought out by Sir Isaac Newton in his famous rotating bucket argument. As Newton saw it, the bucket is rotating in the absolute space and that motion produces the centrifugal forces manifested by the concave shape of the surface of the water in the bucket. The motion of the water is therefore to be considered as "true and absolute," clearly distinguished from the relative motion of the water with respect to the vessel.[20]

Mach, on the other hand, called the concept of absolute space a "monstrous conception,"[21] and claimed that the centrifugal force in the bucket is the result only of the relative motion of the water with respect to the masses in the Universe. Mach argued that if one could rotate the whole Universe around the bucket, the centrifugal forces would be generated, and the concave-shaped surface of the water in the bucket would be identical as in the case of rotating bucket in the fixed Universe. Mach extended this principle to the once famous debate between geocentrists and heliocentrists, claiming that both systems can equally be considered correct.[22]

His arguments, however, remained mostly of a philosophical nature. Since he was a convinced empiricist, he believed that science should be operating only with observable facts, and the only thing we can observe is

[20] Newton, I., 1960, *The Mathematical Principles of Natural Philosophy*, Berkeley CA: University of California Press, pp 10-11.
[21] Mach, E., 1960, *The Science of Mechanics*, 6th ed., LaSalle IL: Open Court, p. xxviii.
[22] *Ibid*., pp. 279, 284.

relative motion. Therefore, every notion of absolute motion or a preferred inertial frame, whether inertial or non-inertial, is not a scientific one but rather a mathematical or philosophical preference.

As Hartman and Nissim-Sabat correctly point out,[23] Mach never formulated the mathematical model or an alternative set of physical laws which can explain the motions of the stars, the planets, the Sun and the Moon in a Tychonian or Ptolemaic geocentric systems. For that reason, some physicists in modern times have tried to "Machianize" the Newtonian mechanics in various ways[24] [25] or even try to construct new theories of mechanics.[26] There have also been attempts to reconcile Mach's principle with the General Theory of Relativity, some of which were profoundly analyzed in the paper by Raine.[27]

In the recent paper[28] we have used the concept of the so-called pseudo-force and derived the expression for the potential which is responsible for it. This potential can be considered as a real potential (as shown by Zylbersztajn,[29] which can easily explain the annual motion of the Sun and planets in the Neo-Tychonian system. In the same manner, one can explain the annual motion of the stars and the observation of the stellar parallax.[30]

It is the aim of this paper to use the same approach to give the dynamical explanation of the diurnal motion of the celestial bodies as seen from the Earth, and thus give the mathematical justification for the validity of Mach's arguments regarding the equivalence of the Copernican and geocentric systems. The paper is organized as follows. In section 2 the vector potential is introduced in general terms. This formalism is then applied to analyze the motions of the celestial bodies as seen from the Earth in section 3. Finally, the conclusion of the analysis is given.

2. Vector potential formalism

Following Mach's line of thought, one can say that the simultaneously rotating Universe generates some kind of gravito-magnetic vector

[23] Hartman, H. I., and Nissim-Sabat, C., 2003, *Am. J. Phys*. 71, 1163–68.
[24] Hood, C. G. 1970, *Am. J. Phys*. 38, 438–442.
[25] Barbour, J., 1974, *Nature* 249, 328–329.
[26] Assis, A. K. T., 1999, *Relational Mechanics*, Montreal: Aperion.
[27] Raine, D. J., 1981, *Rep. Prog. Phys*. 44, 1151–95.
[28] Popov, L., 2013, *Eur. J. Phys*., 34, 383–391, Corrigendum, 2013, *Eur. J. Phys*. 34, 817, Preprint: arXiv:1301.6045.
[29] Zylbersztajn, A., 1994, *Eur. J. Phys*. 15, 1–8.
[30] Popov, L., 2013, arXiv:1302.7129.

potential, **A**. By the analogy with the classical theory of fields[31] one can write down the Lagrangian which includes the vector potential,

$$L = \frac{1}{2}m\dot{\mathbf{r}}^2 + m\dot{\mathbf{r}} \cdot \mathbf{A} + \frac{1}{2}m\mathbf{A}^2 - mU_{ext} \tag{2.1}$$

where m is the mass of the particle under consideration, and U_{ext} is some external scalar potential imposed on the particle, for example, the gravitational interaction.

We know, as an observed fact, that every body in the rotational frame of reference undergoes the equations of motion given by[32]

$$m\ddot{\mathbf{r}} = \mathbf{F}_{ext} - 2m(\boldsymbol{\omega}_{rel} \times \dot{\mathbf{r}}) - m[\boldsymbol{\omega}_{rel} \times (\boldsymbol{\omega}_{rel} \times \mathbf{r})] \tag{2.2}$$

where ω_{rel} is the relative angular velocity between the given frame of reference and the distant masses in the Universe, and $\mathbf{F}_{ext} = -\nabla U_{ext}$ some external force acting on a particle.

It can be easily demonstrated that one can derive Equation (2.2) by applying the Euler-Lagrange equations on the following "observed" Langrangian

$$L_{obs} = \frac{1}{2}m\dot{\mathbf{r}}^2 + m\dot{\mathbf{r}} \cdot (\boldsymbol{\omega}_{rel} \times \mathbf{r}) + \frac{1}{2}m(\boldsymbol{\omega}_{rel} \times \mathbf{r})^2 - mU_{ext} \tag{2.3}$$

By comparison of the general Lagrangian (2.1) and the "observed" Lagrangian (2.3) one can write down the expression for the vector potential **A**,

$$\mathbf{A} = \boldsymbol{\omega}_{rel} \times \mathbf{r} \tag{2.4}$$

It is important to notice that there is no notion of the absolute rotation in this formalism. The observer sitting on the edge of the Newton's rotating bucket can only observe and measure the relative angular velocity between him or her and the distant stars ω_{rel}, incapable of determining whether it is the bucket or the stars that is rotating.

[31] Landau, L. D. and Lifshiz, E. M., 1980, *The Classical Theory of Fields*, 4th ed., Oxford: Butterworth-Heinemann, p. 49.
[32] Goldstein, H., 1980, *Classical Mechanics*, 2nd ed., Addison-Wesley, p. 178.

3. Trajectories of the celestial bodies around the fixed Earth

3.1. Diurnal motion

It is one thing to postulate that rotating masses in the Universe generate the vector potential given by (2.4), but quite another to claim that this same potential can be used to explain and understand the very motion of these distant masses. We will now demonstrate that this is indeed the case.

The observer sitting on the surface of the Earth makes several observations. First, he or she notices that there is a preferred axis (say z) around which all the Universe rotates with the period of approximately 24 hours. Then, according to the formalism given in Section 2, he or she concludes that the Earth must be immersed in the vector potential given by

$$\mathbf{A} = \Omega\hat{\mathbf{z}} \times \mathbf{r} \tag{3.1}$$

where $\boldsymbol{\Omega} \approx (2\pi/24\mathrm{h})$ is the observed angular velocity of the celestial bodies.[33]

One can now re-write the Lagrangian (2.1) together with the Equation (3.1) and focus only on the contributions coming from the vector potential **A**,

$$L_{\mathrm{rot}} = \frac{1}{2}m\dot{\mathbf{r}}^2 + m\Omega\dot{\mathbf{r}} \cdot (\hat{\mathbf{z}} \times \mathbf{r}) + \frac{1}{2}m\Omega^2(\hat{\mathbf{z}} \times \mathbf{r})^2 \tag{3.2}$$

The Euler-Lagrange equations for this Lagrangian, written for each component of the Cartesian coordinates, are given by

$$\begin{aligned} \ddot{x} &= -2\Omega\dot{y} + \Omega^2 x \\ \ddot{y} &= 2\Omega\dot{x} + \Omega^2 y \\ \ddot{z} &= 0 \end{aligned} \tag{3.3}$$

The solution of this system of differential equations reads

$$x(t) = r\cos\Omega t$$

[33] The period of the relative rotation between the Earth and the distant stars is called sidereal day and it equals 23 h 56’ 4.0916”. Common time on a typical clock measures a slightly longer cycle, accounting not only for the Sun’s diurnal rotation but also for the Sun’s annual revolution around the Earth (as seen from the geocentric perspective) of slightly less than 1 degree per day (Wikipedia, 26 Apr. 2013, Sidereal time http://http://en.wikipedia.org/wiki/Sidereal_time).

$$y(t) = r \sin \Omega t \qquad (3.4)$$
$$z(t) = 0$$

where r is the initial distance of the star from the z axis. The observer can therefore conclude that the celestial bodies perform real circular orbits around the static Earth due to the existence of the vector potential **A** given by Equation (3.1). This conclusion is equivalent to the one that claims that the Earth rotates around the z axis and the celestial bodies do not.

3.2. Annual motion

The second thing the observer on the Earth notices is the periodical annual motion of the celestial bodies around the z' axis which is inclined from the axis of diurnal rotation z by the angle of approximately 23.5°. This motion can be explained if one assumes that the Earth is immersed in the so-called pseudo-potential

$$U_{\text{ps}}(\mathbf{r}) = \frac{GM_S}{r_{SE}^2}\, \hat{\mathbf{r}}_{\text{SE}} \cdot \mathbf{r} \qquad (3.5)$$

Here G stands for Newton's constant, M_S stands for the mass of the Sun and $\mathbf{r}_{\text{SE}}(t)$ describes the motion of the Sun as seen from the Earth. The Sun's trajectory $\mathbf{r}_{\text{SE}}(t)$ is shown to be an ellipse in x'-y' plane (defined by the z' axis from the above). Using this potential alone one can reproduce the observed retrograde motion of the Mars or explain the effect of the stellar parallax as the real motion of the distant stars in the x'-y' plane. All this was demonstrated in the previous communications [9, 11].

3.3. Total account

One can finally conclude that all celestial bodies in the Universe perform the twofold motion around the Earth:

i. circular motion in the x-y plane due to the vector potential **A** (3.1) with the period of approximately 24 hours, and

ii. elliptical orbital motion in the x'-y' plane due to the scalar potential U_{ps} (3.5) with the period of approximately one year.

Using Equations (2.1), (3.1) and (3.5) one can write down the complete classical Lagrangian of the geocentric Universe,

$$L = \frac{1}{2} m\dot{\mathbf{r}}^2 + m\Omega\dot{\mathbf{r}} \cdot (\hat{\mathbf{z}} \times \mathbf{r}) + \frac{1}{2} m\Omega^2 (\hat{\mathbf{z}} \times \mathbf{r})^2 - m\frac{GM_S}{r_{SE}^2} \hat{\mathbf{r}}_{\mathrm{SE}} \cdot \mathbf{r} - \mathrm{m}U_{\mathrm{loc}} \qquad (3.6)$$

where U_{loc} describes some local interaction, *e.g.*, between the planet and its moon. It is a matter of trivial exercise to show that these potentials can easily account for the popular "proofs" of Earth's rotation like the Foucault's Pendulum or the existence of the geostationary orbits.

4. Conclusion

We have presented the mathematical formalism which can justify Mach's statement that both geocentric and Copernican modes of view are "equally actual" and "equally correct."[34] This is performed by introducing two potentials: (1) a vector potential that accounts for the diurnal rotations and (2) a scalar potential that accounts for the annual revolutions of the celestial bodies around the fixed Earth. These motions can be seen as real and self-sustained. If one could put the whole Universe in accelerated motion around the Earth, the potentials (3.1) and (3.5) would immediately be generated and would keep the Universe in that very same state of motion *ad infinitum*.

[34] Mach, E. 1960, *The Science of Mechanics*, 6th ed., LaSalle IL: Open Court, pp. xxviii, 279, 284.